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F. M. Hall
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AN INTRODUCTION TO ABSTRACT ALGEBRA

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VOLUME 1

BY

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PREFACE

This work, to be completed in volume 2, is written at a time when abstract algebra is being introduced increasingly into the schools. It attempts to give a broad introduction to the subject, and is intended for those with no previous knowledge of this work but with a fair amount of mathematical sophistication. The level at which the book is written is that of a fairly intelligent sixth-former who wishes to know something about modern algebra and the work that leads up to it, and it could well be read by such a boy before he enters a university. Volume 2 in particular should be useful also for first-year university students, as a general background before a detailed study of the various branches of algebra, while teachers of mathematics who have not studied abstract algebra themselves but who, nevertheless, wish to learn about it and possibly to teach it should find parts of both volumes interesting and useful. While books on groups, rings, vector spaces and the other topics abound, it is not easy to find many which start from the beginning and lead up to the ideas gradually and in a fairly elementary manner. It is hoped that in this the book will satisfy a need.

The present volume leads up to the abstract ideas and methods by means of the study of various particular cases. After a little work on general sets and set theory it deals with the special sets of the integers, other number sets, residues, polynomials and vectors. These should be fairly familiar to the reader, but the emphasis is on those properties that carry over into more general abstract structures, and the proofs selected bear this in mind. Some may wish to omit parts of these chapters as being already known, but some of the results, though important and fairly simple, are not easily available in ordinary text-books.

After a chapter on mappings we study in detail the fundamental laws of algebra, which have lain behind the previous work, and then the final three chapters introduce the theory of groups, give plenty of examples, and study the idea of subgroups as far as Lagrange's theorem.

Volume 2 will continue group theory with a chapter on group homomorphisms and will then introduce elementary ideas in the study of rings, fields and integral domains. Invariant subgroups and ideals will be discussed, and there will be a chapter on vector spaces in which matrices will be mentioned, though no detailed account of matrix theory or linear algebra will be given, since the methods and results of these subjects are, I believe, different in kind to those of abstract algebra proper, being analytic rather than synthetic, more concerned with the properties of individual elements than with the structure as a whole. Volume 2 will end with more detailed work on the algebra of sets and Boolean algebra, and an indication of the main ways in which the work of the whole book is developed into more advanced topics.

Throughout the book I have been careful to give detailed explanations of the reasons for the work, and of the methods used. The technical language has been kept within bounds, as has the symbolism. Yet the work is rigorous as far as it goes, and the notation is in accordance with normal usage, though there is no general agreement in this respect. I have explained new notation as it arises, and occasionally have used my own, as in chapter 6 where residues are printed in bold type. The reader will have nothing to 'unlearn' when he passes on to more advanced text-books.

At each stage I have given as many concrete examples of the structures as I could. It is not always easy to find convincing ones (for example, most simple illustrations of Venn diagrams could be understood just as easily without their aid), and many are taken from other branches of mathematics, but I have done my best and the stock of examples will increase as the subject is taught more and more at an elementary level.

The book is intended to be read with little or no aid from a teacher (not that such aid should be scorned if available) and each chapter ends with a few worked exercises. The exercises themselves are divided into A and B: the first are quite straightforward and should be worked completely. The B exercises are very variable; some are fairly straightforward, others quite difficult and a few give extensions of the bookwork. The reader is not expected to be able to do all these exercises, at least not at a first reading. As with practical examples, so with exercises it is not easy to find those which, without being impossible for any but research workers, are yet non-trivial and worthy of the attention of the student. Here again I have done my best, and here also the stock should increase with use.

I would like to thank some of my former pupils at Dulwich College who read the manuscript and made valuable suggestions. I am indebted to my colleague, Mr D. B. Pennycuick, who read the proofs; and am grateful to the Cambridge University Press for their help throughout all stages of the preparation of the book.

Dulwich College
December 1964

F. M. H.