STATISTICAL PREDICTION ANALYSIS

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Preface

Prediction by its derivation (L. *praedicere*, to say before) means literally the stating beforehand of what will happen at some future time. It is an occupational hazard of many professions: meteorologist, doctor, economist, market researcher, engineering designer, politician and pollster. It is indeed a precarious game because any specific prediction can eventually be compared with the actuality. Many prophets of doom predicting that the world will end at 12.30 on 7 May are left in quieter mood by 12.31. Prediction is a problem simply because of the presence of uncertainty. Seldom, if ever, is it a case of logical deduction; almost inevitably it is a matter of induction or inference. Probabilistic and statistical tools are therefore necessary components of any scientific approach to the formalisation of prediction problems.

In this book we shall be concerned with prediction not only in this narrow sense of making a reasoned statement about what is likely to happen in some future situation but with a much wider class of problems. Any inferential problem whose solution depends on our envisaging some future occurrence will be termed a problem of statistical prediction analysis. The presentation in chapter 1 of a selection of motivating examples illustrates the nature and diversity of statistical prediction analysis, and serves as an introduction to the ingredients of the problem.

A science historian, writing on the development of the concepts and practice of prediction, would probably start by pointing out how primitive man was compelled to attempt prediction, for example the forecasting of the date on which the local river would flood. He might trace how traditions of prediction by magic and sorcery gave way to a realisation that past experience and observation often prove a reliable guide to future events, quoting as an example some well-known folk-lore rhyme about the weather, such as 'A red sky at night is a shepherd's delight'. He would move forward in time to the scientific revolution of the sixteenth and seventeenth centuries highlighting the great advances in descriptive astronomy culminating in what is possibly the greatest predictive achievement of all – the nautical almanack. He might then record the origins of the realisation that there is a relationship between the reliability of a prediction and the inherent variability of the data or the difficulty of accurate measurement, and he could indicate how the science of statistics in its sophisticated present-day form had emerged from such origins. Eventually he might feel the need for a chapter to explain the attitudes of present-day statisticians to the problem of prediction.

His enquiry into such attitudes would soon reveal an interesting though puzzling situation. If his first step were the natural one of taking stock of well-established theory and application of prediction by surveying statistical text-books he would be singularly disappointed. A search of their indexes would reveal only a small minority which listed the term 'prediction' or its near equivalents. Thorough reading of the texts would bring only a little further enlightenment. He would find a few which quoted a 'prediction interval' or a 'confidence interval for a future observation' towards the end of their discussion on regression analysis, where the regression data are to be used to indicate what is likely to happen when the basic experiment is performed at some specified value of the controllable variable. Moreover in very few texts would he find any clear statement of the principles on which such a prediction interval is based and of the method by which the interval is evolved from these principles. He might fare a little better with the small number of texts which present the 'tolerance interval' approach to prediction, but again he would be left with doubts about the basis, interpretation and usefulness of such intervals.

If he broadened his survey to include research literature and specialist books he would certainly find a fuller account of the tolerance interval approach but it is unlikely that all his doubts would be removed. He would also find a well-developed and still developing theory of prediction for stationary processes. On discovering this he might be forgiven for expressing surprise that a more fully developed theory apparently existed for this more complicated situation than for essentially simpler situations. Apart from these expositions he would find only a mixture of *ad hoc* techniques of forecasting by trend curves, exponential weighting, etc. He might then begin to wonder why it is that statisticians have devoted so much time, energy and skill to the fields of estimation, hypothesis-testing and experimental design to the comparative neglect of prediction analysis which is surely at the heart of many statistical applications.

Much of statistical analysis is concerned with making inferences about unknown distribution parameters. For example, given the outcomes from an experiment which is known to be $N(\mu, \sigma^2)$ is it reasonable to suppose that $\mu > 0$; what is a confidence interval for μ? Now the purpose of such inference statements is surely to convey to some second party information about what is likely to happen if the experiment is performed again, or perhaps repeated a number of times. It is surprising therefore that greater thought has not been given to the more direct practical type of inference, where statements are required for what is likely to occur when future experiments are performed. Indeed it is common practice for a statistician first to obtain from the experimental outcome an estimate of the indexing parameter of some class of distributions describing an experiment, and subsequently to use the estimate as if it were the true value to allow prediction. It is paradoxical that while the folly of this approach is pointed out in simple situations there is all too ready acceptance of it in more complicated situations.

This book is an attempt to present certain aspects of statistical prediction theory within a unified framework and notation. As we have already indicated statistical prediction analysis will be here considered in a wide sense, to include any form of statistical analysis where consideration of what may happen (or indeed, to be slightly esoteric at this early stage, what may already have happened) at performances of some future experiment or experiments is essential to the formulation of the problem. The development is considered from both a frequentist and a Bayesian viewpoint for it is our belief that there are situations which are essentially frequentist and other situations which are essentially Bayesian, and the particular type of analysis appropriate to the situation in hand should be used.

It is necessary to provide a comprehensive, yet clear, notation for all the possible distributions involved. Further the notation must not become over-elaborate, for example, when distinguishing between prior and posterior distributions. We believe we have achieved the necessary balance. The notation is introduced as required, mainly in chapters 1, 2 and 3, but a complete list is provided in appendix I.

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