

INDEX

- abelian** group 27
 Abraham, R. 434, 463
 absolute
 angle 166
 determinant 126, 186
 value 34, 47, 178
 Adams, J. F. —6, 197, 239, 284, 420, 463
 addition 20
 adjoint
 of linear map 153, 208
 representation
 of Lie algebra 428
 of Lie group 428
 affine
 approximation 353, 360
 map 77
 space 74
 , normed 289
 of linear complements 138
 structure 74
 subspace 78
 algebra
 , alternative 278
 anti-involution 207
 automorphism 221
 , Cayley 279
 , Clifford 240
 , Dirac 248
 , division 277
 , extensive 274
 , exterior 274
 , fundamental theorem of 47, 390
 , geometric 240
 , Grassmann 274, 275
 , Jordan 436
 , Lie 429, 432
 , linear 67
 , matrix 69
 of complex numbers 48
 of quaternions 176
 , Pauli 248
 , real (linear) 48, 67
 , tensor 241
 alternating group 99, 119
 alternating multilinear map 122
 alternative algebra 278
 analytic 392
 angle 175
 , absolute 166
 , right 147
 annihilator
 , dual 108
 , orthogonal 156, 211
 anti-automorphism 67
 anticommute 179
 anti-involution, algebra 67, 154, 207
 antipodal 169
 antirotation 153
 archimedean 23, 45
 architects 229
 area 126
 arm-twisting 2
 Arnol'd, V. I. 225, 434, 465
 Artin, E. —6, 263, 463
 associative 14
 Atiyah, M. F. —6, 239, 241, 320, 463
 atlas 343, 344
 , admissible 404
 , orientable 432
 , parabolic 231
 , smooth 400
 , standard 224
 atlases, equivalent 345
 automorphism 28, 59, 67
 group 66, 449
 , triality 446
 axis 160

backwards 11
 ball
 , closed 290
 , open 290

- , ping-pong 195
- Banach space 296
- basis
 - of induction 18
 - of linear space 103
- basis, standard 103
 - theorem 104, 135, 158, 212
- between 34
- bijection, bijective 6
- bilinear
 - map 65
 - , continuous 304
 - product 15, 66
- binary
 - expansion 44
 - operation 14
- biquaternion 276
- Bott, R. —6, 241, 284, 420, 463, 464
- bound
 - , lower 39
 - , upper 38
- bounded subset 39, 290, 319
- Boy, W. 433, 463
- Boyer, R. H. —6
- Boy's surface 433
- Bredon, G. E. 454, 465
- Brouwer, L. E. J. 173, 350, 463
- Brown, R. —6, 324, 463
- Bruck, R. H. 284, 285, 463
- bundle
 - , principal 347
 - , tangent 406, 408
 - , vector 410
- Butler, M. C. R. —6
- cancellation** 20
- cardinality 19
- Cartan, É. 435, 456, 465
- cartesian product 12
- Cauchy sequence 43, 295
- Cauchy–Riemann equations 363
- Cauchy–Schwarz inequality 166
- Cayley, A. 278, 463
- Cayley
 - algebra 279, 435
 - chart 234, 236, 426
 - heptagon 283
 - projective line 285
 - projective plane 285, 416
 - triangle 282
- Cayley's theorem 27
- central affine quadric 226
- centre
 - of algebra 67
 - of sphere 168
- chain rule 357
- characteristic of ring 36
 - zero 36
- chart 343
 - , admissible 401
 - , Cayley 234, 236, 426
 - , Lipschitz 260
 - , natural 223
 - , parabolic 231
 - , Pfaffian 263, 349, 426, 427
 - , standard 224
- Chevalley, C. 432, 463
- choice, axiom of 10
- circle group 52, 175
- classification theorem 158, 161, 214
- classroom 5
- Clifford, W. K. 240, 276, 463
- Clifford
 - action 436
 - algebra 2, 174, 240
 - , even 253
 - , universal 247
 - algebras, table of 250
 - embedding 441
 - group 255
 - , projective 255
 - homogeneous space 441
- closed
 - , algebraically 391
 - graph theorem 288
 - map 322
 - subset 291, 316
 - , locally 317
- closure of set 316
- codimension 106
- coefficient system 100
- cofinite topology 311
- Cohen, L. W. 26, 45, 463
- coimage 8
- cokernel 87
- column 116
 - , matrix 68, 103

- commutative
 - diagram 10
 - product 15
 - ring 29
- compact
 - map 322
 - space 319
- companion 448
 - involution 445
- complement
 - , linear 136
 - , orthogonal 156, 211
 - , set 15
- complete 45, 296
- complex
 - differentiable 362
 - field 46
 - group 216
 - , line 216, 225
 - manifold 433
 - number 46
- component 12, 13, 332
- composite, composition 6
- cone, quadric 227
- conjugate, conjugation 46, 177, 252, 279
- connected 41, 329
- constant 6
 - , locally 334
- continuous 292, 313
- contour 8
- contraction lemma 298
- convergence 42, 295
 - , general principle of 45
- convex 81
 - hull 84
- correlated
 - map 211
 - space 151
- correlation
 - , linear 150
 - , semi-linear 201
- correlations, (projectively) equivalent 203, 207
- coset 87
 - space representation 97, 232, 237
- cosine 49, 147
- countable 19
- counterimage 11
- cover, open 317
- critical 372
- Crowell, R. H. 173, 463
- curve 383
 - , Peano 309, 384
 - , smooth 382
- cycle, cyclic 130
- cylinder, quadric 227
- decimal expansion** 44
- degree of polynomial 31
- dense 333
- derivative 361
- Descartes, R. 12
- determinant 121
 - , absolute 126, 186
 - of endomorphism 125
- diagonal of matrix 69
- diagram-chasing 92
- Dieudonné, J. -7, -6, 1, 463
- difference
 - , absolute 34
 - of numbers 20
 - of sets 15
 - vector 74
- differentiable 360
 - manifold 403
- differential 360
 - coefficient 361
 - manifold 403
 - , partial 364
 - , second 393
- differentials, higher-order 392
- dimension 100, 105, 351, 405
- Dirac algebra 248
- direct sum 132
- disconnected 329
 - , totally 332
- discrete topology 312
- dishonesty 249, 251
- dissimilar 194
- distance 41, 146, 164, 290
- distributive 20, 29
- divide 20
- division algebra 277
- divisor 20
 - of zero 30
- domain 5
 - , invariance of 350

478

dual
 , continuous linear 302
 exact sequence 92
 , linear 184
 of semi-linear map 201
 tangent bundle 434
 dynamics, Hamiltonian 434

Ehrlich, G. 26, 45, 463
 ellipse 228
 ellipsoid 228
 elliptic paraboloid 229
 embedding
 , Clifford 441
 , smooth 412
 , topological 315
 endomorphism 59
 endomorphism algebra 66
 epimorphism 59, 99
 Epstein, D. B. A. 443, 454, 465
 equation of quadric 225
 equivalence 13
 class 14
 euclidean
 , locally 343
 space 167
 even
 Clifford algebra 253
 permutation 119
 everywhere free 419
 exact
 pair 89
 sequence 90, 96
 , dual 92, 107
 , short 91
 , split 95, 97
 exponential map 48
 exponentiation 21
 extensive algebra 274
 exterior algebra 274

factor 20
 factorial 22
 fibre 8
 bundle, principal 347
 field 36
 , algebraically closed 391
 , double 132

INDEX

, involuted 251
 , tangent vector 419
 finite
 , locally 334
 number 17
 set 19
 finite-dimensional 100, 112
 five lemma 98
 fixed field 251
 Flett, T. M. —6
 form
 , hermitian 202
 induced by correlation 201
 , quadratic 148
 forwards 11
 four lemma 98
 Fox, R. H. 173, 463
 frame 116
 , basic 116
 framing 116
 , basic 116
 , elementary 117
 , unimodular 117
 free 101, 135, 419
 subset, maximal 102
 vector 75
 Freudenthal, H. 456, 465
 Frobenius' theorem 284
 Fuchs, L. 48, 463
 functional dependence 390

Gauss, K. F. 173, 463
 general linear group 106
 generation
 of algebra 112
 of ring 112
 geometric algebra 240
 geometrical model 41
 geometry 41
 , projective 144, 216
 , topological —7, 1
 Gilmore, R. 432, 455, 465
 Gleason, A. M. 427, 463
 Goffman, C. 288, 463
 good correlation 204
 gradient, absolute 300
 graph 13
 Grassmann, H. 274, 463
 Grassmann algebra 274, 275

- Grassmannian 140, 223, 275
 , quadric 225, 425
 , semi-neutral 225
 , tangent space of 411
 great sphere 169
 greater than 22
 greatest 23
 group 26
 , abelian 27
 action 97, 435
 , additive 27
 , alternating 99, 119
 , Clifford 255
 , general linear 106
 , Lie 421
 , Lorentz 427
 map 27
 of permutations 26
 , orthogonal 153, 216
 , Pin 255
 reversing map 27
 , Spin 255, 264
 structure 26
 , symplectic 216
 , topological 336
 , unimodular 124
 , unitary 216
 Gruenberg, K. W. 144, 463
- Hahn–Banach theorem** 288
 half-spinor space
 , complex 251
 , real 249
 Halmos, P. R. 4, 19, 309, 463
 Hamilton, W. R. 176, 463
 Hamilton triangle 281
 Hamiltonian dynamics 434
 Hampson, A. —6
 Hausdorff space 320
 Heine-Borel theorem 318
 Heisenberg uncertainty principle 309
 Helgason, S. 432, 464
 heptagon, Cayley 283
 Hostenes, D. 248, 464
 hermitian
 form 202
 product 202
 Hilbert, D. 427, 464
 Hilbert's fifth problem 427
 Hirzebruch, F. 347, 410, 464
 homeomorphism 313
 , locally a 333
 , smooth 373, 375
 homogeneous space 338
 homomorphism 28
 honesty 276
 Hopf, H. 197, 464
 Hopf map 144, 197, 285, 437
 Horrocks, G. —6
 hull, convex 84
 Hurewicz, W. 85, 350, 384, 464
 Hurwitz, A. 284, 420, 464
 Hurwitz' theorem 284
 hyperbola 228
 hyperbolic 149, 194
 paraboloid 229
 plane 153, 213
 hyperboloid 228
 hyperplane 106
 at infinity 142
 reflection 158
- ideal** 31, 89
 , left 70
 , minimal left 71, 113
 , right 70
 idempotent 193
 identification topology 314
 identity map 7
 image 5, 8, 11
 , inverse 11
 , linear 101
 imaginary part 47
 immersion 413
 implicit function theorem 380
 inclusion 8
 increment 74
 formula 375
 index 161, 214, 475
 , Witt 162
 induction, mathematical 18
 infimum 39
 infinite 19
 , countably 19
 infinity 42, 142
 injection, injective 5
 integer 35

480

INDEX

- intermediate-value theorem 330
 intersection 15
 interval 42
 inverse 15
 function theorem 378
 inversion 305, 369
 invertible 150
 involute 181, 252
 involuted field 251
 involution 59
 , algebra 67
 , companion 445
 , main 181, 252
 irreducible
 automorphism 193
 semi-linear map 200
 Irwin, M. C. -6
 isolated critical point 396
 isomorphism 28, 59, 313
 isotropic
 correlated space 210
 orthogonal space 149
 isotropy subgroup 98, 339, 437
- Jacobi** identity 432
 Jacobian matrix 362
 Jacobson, N. 455, 465
 James, I. M. 416, 443, 464, 465
 join, affine 109
 Jordan algebra 436
- Kaplansky**, I. 432, 464
 Kelley, J. L. 4, 464
 kernel 29, 61, 151
 rank 110
 Kervaire, M. 284, 420, 464
 Kleinfeld, E. 284, 464
- Lagrange** multipliers 387
 Lagrangian subspace 225
 Lang, S. 381, 464
 least 23
 left
 coset 87
 linear space 183
 module 187
 left-coset exact 97, 218
- left-handed 129
 length 146, 164
 less than 22
 level 8
 Lie, S. 421, 464
 Lie
 algebra 429, 432
 , semi-simple 455
 , simple 455
 bracket 429
 group 421
 map 421
 , semi-simple 455
 , simple 455
 limit 42, 295, 317
 line
 , affine 81, 105
 , Cayley projective 285
 complex 216, 225
 , intuitive 41
 , linear 105
 , projective 140
 linear
 algebra 67
 bundle 410
 complement 136
 correlation 150
 dependence 100, 135
 dual 64, 65, 100, 184
 , continuous 302
 image 101
 independence 101
 map 57
 , continuous 294, 300
 over double field 135
 maps, equivalent 111
 part 77
 product 63
 section 59, 95, 138
 space 55
 , normed 289
 over double field 135
 structure 55
 subspace 60
 sum 61
 union 61
 linearly free 101, 135
 line-segment 81, 146
 linked spheres 173, 197
 Lipschitz, R. 260, 465

- Lipschitz chart 260
 locally
 closed 317
 constant 334
 euclidean 343
 finite 334
 trivial 326
 Loos, O. 449, 457, 465
 Lorentz group, proper 161, 268, 427
- MacLane, S.** 85, 464
 main involution 181, 252
 manifold
 , analytic 401
 , complex 433
 , differentiable 403
 , differential 403
 , infinitely smooth 401
 , orientable 432
 , smooth 401
 , topological 343
 map 5
 , inverse 7
 , invertible 7
 Mather, J. N. 434, 464
 matrix 68
 maximum, local 372, 386
 mean value theorem 377
 membership 4
 Milnor, J.W. -6 284, 403, 415, 420,
 464
 minimum, local 372, 386
 model
 , geometrical 41
 , linear 59
 , topological 313
 module 71, 187
 over double field 134
 modulo 23
 modulus 47
 monomorphism 59, 98
 Montgomery, D. 427, 461, 464, 465
 morphism 28
 Morton, H. R. 454
 Moufang identities 287
 Moufang, R. 287, 465
 multilinear map 66
 multiplication 20
- natural**
 chart 223
 number 17
 negative 33
 of a correlated space 211
 negative-definite 149, 215
 neighbourhood 290, 312
 Neumann, J. von 4
 neutral
 correlated space 210
 orthogonal space 149
 Newns, W. F. 390, 412, 436, 465
 non-associative 29, 282
 non-degenerate
 affine quadric 225
 correlation 151, 203
 critical point 396
 orthonormal subset 243
 norm 47, 164, 178, 259, 288, 300
 , product 289
 , quadratic 289
 , sum 289
 normal subgroup 88
 normed
 affine space 289
 division algebra 277
 linear space 289
 norms, equivalent 292, 299
 North pole 168
 nullity 110
 number
 , complex 46
 , finite 17
 , natural 17
 , prime 50
 , rational 37
 , real 40
- octaves** 278
 octonions 278
 odd permutation 119
 one 5
 open
 cover 317
 map 322
 mapping theorem 288
 section 323
 set 291, 311
 orbit 98

482

order 22
 ordered field 36
 orientable
 atlas 432
 smooth manifold 432
 orientation 129
 oriented linear space 129
 origin 56
 orthogonal 149, 211
 annihilator 156, 211
 complement 156, 211
 decomposition 156
 group 153, 216
 isomorphism 152
 map 152
 , special 153
 space 149
 , neutral 149, 163
 orthonormal
 basis 158, 212
 subset 158, 242

Paige, L. J. 436, 465
 pair, ordered 12
 parabola 228
 parabolic
 affine quadric 227
 atlas 231
 chart 231
 paraboloid 229
 parallel 76
 parallelizability 420
 parameter space 383
 parametric representation 333
 parity of permutation 119
 partial differential 364
 coefficients 362
 partition 9
 path 2
 path-connected 334
 Pauli algebra 248
 Peano, G. 384
 Peano curve 309, 384
 Pedrick, G. 288, 463
 pentagon, rebracketing 7, 283
 period, periodic 49
 periodicity 249, 270
 permutation 6
 , even 119

INDEX

 , odd 119
 , parity of 119
 , sign of 119
 Pfaffian 260
 chart 260, 349, 426
 , complete 261
 phase space 434
 Phillips, A. 433, 464
 Pin group 255, 264
 ping-pong ball 195
 pivotal condensation 131
 plane
 , affine 105
 , Cayley projective 285
 , hyperbolic 149, 153, 210, 213
 , intuitive 41, 146
 , linear 105
 , projective 140
 , symplectic 211, 213
 point 56
 at infinity 141
 , projective 140
 , regular 135
 polynomial 31
 Pontryagin (Pontrjagin), L. S. 173,
 427, 432, 464, 465
 positive 33
 number 18
 positive-definite 149, 164, 215
 power 21, 26
 set 8
 predecessor 17
 prime number 50
 primitive idempotent 193
 product
 , algebra 67
 , bilinear 15, 66
 induced by correlation 201
 , matrix 69
 norm 289
 of groups 29
 of numbers 20, 21
 of rings 30
 of sequence 20
 of sets 12
 on a set 14
 , scalar 148
 , tensor 187, 189
 topology 325
 , scalar triple 179, 280

- , vector 178
- projection 13
 - , orthogonal 146
 - , smooth 412
 - , tangent 408
 - , topological 315
- projective
 - Clifford group 255
 - correlation 206
 - geometry 144, 216
 - line 140
 - , Cayley 285
 - map 143
 - plane 140
 - planes, embeddings of 415
 - quadric 225
 - quadric, complex 238
 - space 140, 223
 - subspace 143
- proper map 324
- pure
 - part 47
 - quaternion 176
- Pythagoras' theorem 147

- quadratic**
 - form 148
 - norm 289
- quadric
 - , affine 225
 - cone 227
 - cylinder 227
 - Grassmannian 225, 425
 - , affine 225
 - , semi-neutral 225
 - , projective 225, 435
 - quasi-sphere 173, 217, 340, 423
 - quaternion 176
 - product 176
 - quaternionic dimension 185
 - quotient
 - group 88
 - , linear 79, 85
 - of numbers 37
 - of set 9
 - space 314
 - topology 314
- radial** tangent vector 414
- radius 168
- Radon, J. 420, 465
- Radon–Hurwitz sequence 259, 273, 420
- rank 109, 151
 - theorem 390
- rational
 - field 37
 - number 37
- real
 - field 40
 - number 40
 - part 47
- rebracketing pentagon 7, 283
- recursive definition 19
- reducible
 - automorphism 193
 - semi-linear map 200
- reflection 158
- reflexive correlation 202
- regular point 135
- relativity 149, 268
- remainder 23
- restriction 10
- reverse 181, 252
- reversion 181, 252, 254
- Riemann representation 196
- right angle 147
 - linear space 183
 - module 187
- right-handed 129
- ring 29
 - map 30
 - of integers 34
 - of polynomials 31
 - of remainders 30
 - , ordered 33
- ring-reversing map 31
- Robertson, S. A. –6
- rotation 146, 153
- rotations
 - of \mathbf{R}^3 2, 160, 180
 - of \mathbf{R}^4 182
- Russell, B. 24, 465
- Russell paradox 4, 24

- Samelson, H.** 461, 465
- scalar 56

- multiplication 55
 - product 148
 - triple product 179, 280
- Schröder–Bernstein theorem 24
- section 10
 - , linear 95
 - , open 323
 - , zero 409
- Segre, B. 48, 465
- self-adjoint 208
- semi-linear map 198
- semi-neutral 225
- semi-orientation 161
- sequence 18
 - , Cauchy 43, 295
 - , convergent 42, 295
 - , doubly-infinite 35
 - , exact 90
 - , finite 19
- sesqui-linear 201
- set 4
 - , empty 4
 - , finite 19
 - , infinite 19
 - , null 4
- sets
 - ; disjoint 4
 - , intersecting 4
 - , overlapping 4
- Shapiro, A. –6, 241, 463
- short exact sequence 91
- side of hyperplane 128
- signature 161, 213
- similar 112, 194
- Simmons, G. F. 288, 384, 465
- sine 49
- singular 371
- singularity 372
- skew 73
 - correlation 201
- skew-adjoint 208
- Skornyakov, L. A. 284
- smooth 360, 392
 - curve 382
 - , infinitely 382, 392
 - manifold 401
- smooth map 403
 - structure 401
 - submanifold 392
 - subset 381
- surface 382
- solution 5
- soup plate 2
- source 5, 39
- South pole 168
- spanning subset 101
 - , minimal 102
- sphere 168, 290
 - , seven-dimensional 403
 - , unit 168
 - , zero 159, 168
- Spin group 255, 264
- spinor space
 - , complex 251, 276
 - , real 249, 276
- split exact sequence 95, 97
- Spurgeon, C. J. 454
- square 26, 148
 - , a 195
 - , commutative 15
 - root 42
 - , twisted 221
- stability of maps 434
- stabiliser 98
- standard
 - atlas 224
 - basis 103
 - chart 224, 399
 - hyperbolic plane 149, 210
 - symplectic plane 211
 - topology 311, 312
- Steenrod, N. E. 420, 443, 454, 465
- Stiefel manifolds 345, 443, 465
- step, inductive 18
- stereographic projection 169
- Study, E. 276, 435, 459, 466
- subgroup 28
 - , normal 88
- submanifold, smooth 382, 404
- submersion 414
- subring 31
- subset 8
 - , free 101
 - , proper 8
 - , spanning 101
- subspace, topological 314
- subtraction 56, 74
- successor 17
- sum 20, 21
 - norm 289

INDEX

485

- supremum 38
 surface, smooth 382
 surjection, surjective 5
 symmetric correlation 201
 symmetry of second differential 393
 symplectic 217
 group 216
 plane 211, 213

tangency 353
 tangent 361
 bundle 406, 408
 , dual 434
 map 406, 409
 projection 408
 space 75, 381, 408
 vector 75, 408
 field 419
 along a map 433
 target 5
 Taylor series 392
 Taylor's theorem 397
 ten 24, 207, 270
 tensor
 algebra 241
 product of algebras 187
 product of linear spaces 189
 term
 , leading 31
 of matrix 69
 of sequence 18
 three 312
 Tits, J. 459, 466
 topological
 embedding 315
 group 336
 manifold 343
 projection 315
 space 312
 topology 311
 , cofinite 311
 , discrete 312
 , identification 314
 , induced 314, 343
 , product 325
 , quotient 314
 , subspace 314
 , trivial 311
 torus 350, 414

 trace 367
 transformation 6
 transitive group action 98, 338, 435
 transitivity 23
 translate 77
 translation 76
 transposition 70
 transversal intersection 433
 triality 435
 automorphism 446
 , principle of 456, 459
 , quadric 457
 triangle 84
 axiom 74
 , Cayley 282
 , Hamilton 281
 inequality 165, 288
 trichotomy 22
 trilinear 179
 trivial
 map 326
 topology 311
 tuple
 , n - 19
 , 2- 12
 twenty-nine 312
 twisted square 221
 two 5
 type of orthonormal subset 243

ultramundane correspondent 260,
 465
 uncertainty principle 309
 unimodular 117
 group 124
 union 15
 unit quaternion 178
 unitary
 group 216
 space 171, 213
 unity 14
 universal Clifford algebra 247
 upper bound axiom 40

value 5
 , absolute 34, 47, 178
 vector 56, 74
 , bound 75

486

bundle 410
 field, tangent 419
 field problem 239
 , free 75
 product 178
 space 56, 74
 , tangent 75, 408
 vertex of cone 227

Walker, A. G. 412, 465
Wall, C. T. C. -6, 271, 465
Wallman, H. 350, 384, 464
Weir, A. J. 144, 463
 well-ordered 23
Weyl, H. 217, 432, 465

INDEX

wheel 130
Whitney, H. 415, 465
Willmore, T. J. -6
Witt, E. 173, 465
Witt
 decomposition 162, 214
 index 162

Y space 321, 343

zero 4, 56
 , divisor of 30
 section 409
Zippin, L. 427, 464