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Ian R. Porteous
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TOPOLOGICAL GEOMETRY

Topological Geometry

Second Edition

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FOREWORD

Mathematicians frequently use geometrical examples as aids to the study of more abstract concepts and these examples can be of great interest in their own right. Yet at the present time little of this is to be found in undergraduate textbooks on mathematics. The main reason seems to be the standard division of the subject into several watertight compartments, for teaching purposes. The examples get excluded since their construction is normally algebraic while their greatest illustrative value is in analytic subjects such as advanced calculus or, at a slightly more sophisticated level, topology and differential topology.

Experience gained at Liverpool University over the last few years, in teaching the theory of linear (or, more strictly, affine) approximation along the lines indicated by Prof. J. Dieudonné in his pioneering book *Foundations of Modern Analysis* [14], has shown that an effective course can be constructed which contains equal parts of linear algebra and analysis, with some of the more interesting geometrical examples included as illustrations. The way is then open to a more detailed treatment of the geometry as a Final Honours option in the following year.

This book is the result. It aims to present a careful account, from first principles, of the main theorems on affine approximation and to treat at the same time, and from several points of view, the geometrical examples that so often get forgotten.

The theory of affine approximation is presented as far as possible in a basis-free form to emphasize its geometrical flavour and its linear algebra content and, from a purely practical point of view, to keep notations and proofs simple. The geometrical examples include not only projective spaces and quadrics but also Grassmannians and the orthogonal and unitary groups. Their algebraic treatment is linked not only with a thorough treatment of quadratic and hermitian forms but also with an elementary constructive presentation of some little-known, but increasingly important, geometric algebras, the Clifford algebras. On the topological side they provide natural examples of manifolds and, particularly, smooth manifolds. The various strands of the book are brought together in a final section on Lie groups and Lie algebras.

Acknowledgements

I wish to acknowledge the lively interest of my colleagues and students

in the preparation of this book. Particular thanks are due to the students of W3053 (Advanced Calculus) at Columbia, who met the book in embryo, and of BH (Algebra and Geometry), CH (Linear Algebra and Analysis) and DP5 (Lie Groups and Homogeneous Spaces) at Liverpool, who have suffered parts of it more recently. I owe also a considerable debt to Prof. T. J. Willmore, who was closely associated with the earliest drafts of the book and who shared in experiments in teaching some of the more elementary geometrical material to the BH (first-year) class. Various colleagues—including Prof. T. M. Flett, Prof. G. Horrocks and Drs. R. Brown, M. C. R. Butler, M. C. Irwin and S. A. Robertson—have taught the 'Dieudonné course' at Liverpool. Their comments have shaped the presentation of the material in many ways, while Prof. C. T. C. Wall's recent work on linear algebra over rings (for example [57]) has had some influence on the final form of Chapters 11 and 13.

The linear algebra in the first half of the book is fairly standard, as is the treatment of normed linear spaces in Chapter 15 and of topological spaces in Chapter 16. For most of Chapters 9 and 11 my main debt is to Prof. E. Artin's classic [3]. My interest in Clifford algebras and their use in relativity was stimulated by discussions with Dr. R. H. Boyer, tragically killed in Austin, Texas, on August 1st, 1966. Their treatment here is derived from that of M. F. Atiyah, R. Bott and A. Shapiro [4], while the classification of the conjugation anti-involutions in the tables of Clifford algebras (Tables 13–66) is in a Liverpool M.Sc. thesis by A. Hampson. The observation that the Cayley algebra can be derived from one of the Clifford algebras I also owe to Prof. Atiyah. Chapters 18 and 19, on affine approximation, follow closely the route charted by Prof. J. Dieudonné, though the treatment of the Inverse Function Theorem and its geometrical applications is from the Princeton notes of Prof. J. Milnor [42]. The proof of the Fundamental Theorem of Algebra also is Milnor's [44]. The method adopted in Chapter 20 for constructing the Lie algebras of a Lie group was outlined to me by Prof. J. F. Adams.

Finally, thanks are due to Mr. M. E. Matthews, who drew most of the diagrams, and to Miss Gillian Thomson and her colleagues, who produced a very excellent typescript.

References and Symbols

For ease of reference propositions and exercises are numbered consecutively through each chapter, the more important propositions being styled theorems and those which follow directly from their immediate predecessors being styled corollaries. (Don't examine the system too closely—there are many anomalies!)

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Implication is often indicated by the symbol ⇒ or ⇐, the symbol ⇔ being an abbreviation for 'if, and only if'.

The symbol □ is used to mark the end of a proposition or exercise and such proof or hints at proof as may be given.

Numbers within [ ] are references to the bibliography on pages 463–466. The entries in the bibliography are very varied in character. Some are texts which are readily accessible and which complement the material of this book. Others are given because of their historic interest.

Following the bibliography there is a list of the more important mathematical symbols used in the text, as well as a comprehensive index.

Liverpool, September 1969

IAN R. PORTEOUS

The opportunity has been taken in this second edition to correct a number of misprints and minor errors, some brought to my attention by readers, to all of whom I am most grateful.

The text remains essentially unaltered, the earlier chapters providing a route from first principles through standard linear and quadratic algebra to geometric algebra—the study of the classical matrix groups and their homogeneous spaces, Grassmannians, quadrics and the like—with Clifford's geometric algebras taking pride of place. In parallel with this is an account, again from first principles, of the elementary theory of topological spaces and of continuous and differentiable maps leading up to the definitions of smooth manifolds and their tangent spaces and of Lie groups and Lie algebras. Here the geometric algebra provides numerous significant examples. It is the study of these examples, using topological and differentiable techniques whenever necessary, which we call 'topological geometry'.

The main addition to the book is a new chapter, Chapter 21, on triality, a feature of the group Spin 8 which illuminates the structure of several of the other Spin groups and which is related to a property of six-dimensional projective quadrics first noticed eighty years ago by Study in work on the rigid motions of three-dimensional space. This chapter leads on naturally from Chapter 13 on Clifford algebras and Chapter 14 on the Cayley algebra as well as from Chapter 20 with its final section on Lie groups and Lie algebras. There is plenty of interest in the details, which include a number of important transitive group actions and a description of one of the exceptional Lie groups, the group G_2. Much of this material is difficult to find elsewhere.

Liverpool, September 1979

IAN R. PORTEOUS