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978-0-521-29750-9 - A Handbook of Numerical and Statistical Techniques: With Examples
Mainly from the Life Sciences

J. H. Pollard

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