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978-0-521-29750-9 - A Handbook of Numerical and Statistical Techniques: With Examples

Mainly from the Life Sciences

J. H. Pollard

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**A handbook of numerical and statistical techniques
with examples mainly from the life sciences**

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A HANDBOOK OF

Numerical and statistical techniques

WITH EXAMPLES MAINLY FROM
THE LIFE SCIENCES

J. H. POLLARD

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Preface

Although scientists nowadays have ready access to large-scale computer systems which incorporate sophisticated package programs for solving statistical and numerical problems, many of their day-to-day problems can be solved (and are possibly better solved) on the small programmable desk calculators and mini-computers now finding their way into every laboratory. These machines, with capacities not unlike those of computers two decades ago, are proving popular because of their ready availability, their cheapness and the fact that the research worker can ‘play around’ with his data as long as he likes. His final analysis is likely to be better than that produced by batch processing on a large computer. Package programs are available for these desk machines, but the research worker usually needs to tailor these programs to meet his own particular needs. This handbook is designed to aid him in this task.

Package programs on large-scale computer systems solve standard problems and they usually produce copious output. The scientist needs to understand something of the program methodology and he should be familiar with the various items output by the program. This is particularly true if he wants to modify or combine packages to solve non-standard problems. Many of the basic numerical and statistical techniques are described in this handbook.

Interactive computer terminals are now becoming more widely available. They have the advantage of the full-scale computer system and the convenience of a desk-top programmable calculator. The comments made in respect of programmable calculators and full-scale computers also apply to the interactive terminal.

A word of caution should be given. It is dangerous to accept package programs in blind faith. Write-ups can be misleading and programs are sometimes found to be faulty. The user is advised to test with known data any program he has not used previously.

This handbook is designed for ready reference by the scientist who has

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some knowledge of statistical method. It is not a textbook, although some of the techniques (for example, least squares) are explained in considerable detail, and exercises are given at the end of each chapter to amplify certain points. Each technique is demonstrated by at least one example. When he wants to solve a problem, the practical scientist will try a reasonable method. If it does not work, he will try another. One of them will usually work. With more difficult numerical and statistical problems he is advised to seek professional assistance.

The book is divided into three parts. Part I deals with numerical techniques and Part II statistical techniques. Part III is devoted to the method of least squares, which can be regarded as both a numerical technique and a statistical method.

Most of the writing of this book was done while on study leave in Europe during 1975–76, and I would like to thank Professors Gustav Feichtinger, Jan Hoem and Peter Whittle for their hospitality in Vienna, Copenhagen and Cambridge respectively. I would also like to thank Dr Odd Aalen, Dr Joe Gani, Mr Cameron Kirton, Dr Søren Johannsen and Professor Mark Williamson for their many helpful comments, and Miss Betty Thorn for drawing the diagrams. The book is dedicated to my wife who showed great patience during the many evenings when I was busy writing.

Macquarie University, Sydney
December 1976

J.H.P.