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Siferu Mizohata
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Preface

This book is based upon my lectures given at Kyōto and other universities. The original lectures covered some modern developments of the theory of partial differential equations, and the preliminaries which constitute chapters 1 and 2 in this book were merely outlined in the actual lectures. The reason why these rather lengthy chapters have been included is that the book was written not only for future analysts, but also for other unprepared young scientists or engineers who have a keen interest in the subject but little time for wading through existing copious literature merely to acquire basic knowledge sufficient to start reading the main body of this book.

As early as Autumn 1961, Professor S. Iyanaga, together with Professor K. Yosida, suggested that I should write a book of this kind. Since that time, I have tried several times to set down a workable arrangement without success. One early plan was to divide the contents of the book into two parts, and deal with the classical account in the first part, then proceed to the modern account of the same subjects in the second. This seemingly simple arrangement was in fact difficult because of the choice of subject-matter and the style of presentation; if the classical part, which might well be a foundation of the modern theory, were to be written without sufficient underlying notion against a certain historical background, it would, perhaps, be tedious for the reader.

For this reason, I introduced the modern treatment at the outset and then explained some classical problems in the light of recent developments. It is true to say that since the publication of L. Schwartz's monumental *Théorie des Distributions* (1950–1) a great deal of work has been carried out by many mathematicians using the new method. But, in actual fact, most of them were generalizations of problems which had already been (their rigour apart) treated before the publication of Schwartz's book by some other (or the same) mathematicians, but under restricted conditions. Aware of this situation, I have attempted to clarify wherever possible the basic ideas of comparatively old problems.

As for the organization of the contents, each chapter was written so as to be as self-contained as possible, assuring the individual reader's choice of access according to his interest and mathematical maturity. Various function spaces which appear in chapter 2 may not be familiar to a beginner, so that it is absolutely necessary for such readers to refer back to the definitions of these spaces from time to time until the ideas are truly grasped. For the reader's convenience, a list of symbols with simplified definitions, including these function spaces, is given at the end of the book.

In chapter 3, I deal with boundary value problems for elliptic partial differential equations. Relatively simple equations were listed to clarify the basic problems in this genre, where $\mathscr{D}_{L^2}{}^1(\Omega)$ and $\mathscr{E}_{L^2}{}^1(\Omega)$ play important rôles. Note that these spaces are both complete so that they are, of course, Hilbert spaces.

For example, in the last chapter of Courant–Hilbert: *Methoden der mathematischen Physik II*, $\mathscr{E}_{L^2}{}^1(\Omega)$ is defined as the completion by the metric of $\|u(x)\| + \sum \|\partial u(x)/\partial x_i\|$ of the function space of continuously differentiable functions in $\bar{\Omega}$. On the other hand, in this book, the same space is defined as a subset of the elements of $L^2(\Omega)$ of which all first order partial derivatives in the sense of distribution again belong to $L^2(\Omega)$, or in short are 'once differentiable within L^2'.

Some properties of the functions of this kind are given in chapter 2; more detailed properties are studied in chapter 3. The knowledge of Lebesgue integral is indispensable in chapter 3, but the elementary theory is sufficient. After reading chapter 3, the reader should glance at chapter 8 to study Green's functions and Green's kernels, and acquaint himself with the historical background of the theory established in chapter 3.

In chapter 4, starting from classical results, the idea 'hyperbolic type' is explained. This was a creation of Hadamard who used it to classify equations in a more intrinsic sense than his predecessors. One of the significant results of using his notion is that, given an initial value of C^∞-function for an elliptic partial differential equation, the initial value problem is not soluble in general without imposing appropriate conditions on the value.

Chapter 5 is devoted to the existence theorem of the solution of a general evolution equation. The alert reader may notice that this is entirely different from the Cauchy–Kowalewski existence theorem in its nature, illustrating an intrinsic difference between ordinary differential equations and partial differential equations.

Chapter 6 and chapter 7 introduce boundary value problems for hyperbolic equations with variable coefficients.

Finally, in 'Supplementary remarks', a brief sketch of the general boundary value problem for elliptic equations is given. If we wish to insist on the principle that completed theory must be stated in the light of a generalized form, this should be suitably incorporated in chapter 3, but I thought this was a little abrupt for the beginner, and also for the student of theoretical physics.

The bibliography at the very end gives references and suggestions for the interested reader who wishes to explore further in various genres.

On the whole, the approach is biased in favour of the method of functional analysis, starting from a rather abstract setting, then step by step, revealing interesting aspects of the theory. It is the author's hope that in this way the book communicates to the reader the beauties which are characteristic of the theory of partial differential equations. However, the author fears that it may be inordinate to expect the reader to appreciate the theory merely by reading a single volume with a limited scope such as this. As to style, it should be mentioned that, merely due to my taste, the proofs of the facts which are employed in this book are made as orthodox as possible, despite the widely known existence of more refined methods. As for the names of the established facts, the facts which are fundamental to future theories are stated under the title of 'lemmas', and finalized versions are collected under the title of 'theorems'.

During the preparation of this book, my colleagues, Mr Masaya Yamaguti and others, made various comments and gave invaluable advice related to the contents. I thank them all for their valuable criticisms. I should also like to express my thanks to Professor S. Iyanaga and Professor K. Yosida who really gave me the chance to write this book as a new title in the celebrated Iwanami series. My gratitude also goes to Mr Eiji Negishi, Mr Masahisa Makino, Mr Hideo Arai, who are editorial staff of Iwanami Shoten, for their efficient cooperation, and especially, to Mr Makino who, since the autumn of 1961, generously assisted me with a number of matters relating to the manuscript.

Sigeru Mizohata

May 1965
Kyōto

Preface to the English edition

I am very pleased to learn that *Theory of Partial Differential Equations* will appear in English.

As I have already said in the preface of the original Japanese edition, the intention of the book is to provide the prospective reader with recently developed methods centering around 'distribution' which have been extensively used in the fundamental theory of partial differential equations.

Although, remarkable progress has been made in various directions since the book was published in 1965 – especially in boundary value problems and the existence theory of solutions of general equations – I believe that the book still presents useful up-to-date information on some aspects of this fascinating mathematical discipline.

Finally, I am indebted to Mr Katsumi Miyahara for his painstaking translation of the book.

Kyōto University, Japan
September 1972 Sigeru Mizohata