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MODAL LOGIC: AN INTRODUCTION

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BRIAN F. CHELLAS

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To the memory of my mother

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PREFACE

This book is an introductory text in modal logic, the logic of necessity and possibility. It is intended for readers with the equivalent of a first course in formal logic, and it is designed to be used as a basic text in courses at the advanced undergraduate or beginning graduate level. The material in the book can easily be covered in a full-year course; with selectivity most of the material can be covered in a single term.

There are three parts to the book. Part I consists of two chapters, meant to introduce the reader to the subject of modal logic and to furnish a sufficient background for the parts that follow. Chapter 1 is a relatively informal examination of *S5*, one of the best-known systems of modal logic. Chapter 2 – 'Logical preliminaries' – contains almost everything needed for an understanding of the rest of the book. Some readers may prefer to go quickly through this chapter and then reread as necessary sections required in the context of succeeding chapters.

Part II comprises four chapters on standard models and normal systems of modal logic. The models, sometimes called 'Kripke models', are explained in chapter 3. In chapter 4 normal systems are presented from an axiomatic standpoint. Chapter 5 contains theorems on completeness and decidability, which bring together the model-theoretic and deductive-theoretic treatments of the preceding chapters. As an illustration of normal systems chapter 6 offers a discussion of deontic logic, the logic of obligation.

Part III is patterned like its predecessor, but here the topics are minimal models and classical modal logics. Thus chapter 7 is about the models (also known as 'neighborhood' or 'Scott-Montague' models), chapter 8 is an axiomatic account of the logics, and chapter 9 deals with completeness and decidability. Chapter 10 presents conditionality and (again) obligation by way of example.

An important feature of the book is the exercises that follow the sections of the chapters. These have been constructed both to consolidate understanding of the preceding material and to anticipate subsequent developments. They are an integral part of the text, and I have high hopes that the reader will attempt them as they appear.

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I have appended to the text a short bibliography citing most of the works I found useful in writing this book. Many of these books and articles will take the reader farther afield to topics and results not treated here, and several contain good bibliographies.

I have a number of debts to record. First among these is to Lee Bowie, who several years ago suggested that we author a textbook in modal logic – I to write the chapters on propositional modal logic, he to write on quantification, identity, naming, and description. When it later became apparent that the material on propositional modal logic was bulky enough to warrant separate publication, Bowie graciously encouraged me to proceed alone.

In this connection I also want to express my gratitude to Richard Jeffrey and David Lewis, for their advice and support, and for recommending my project to Cambridge University Press and its distinguished editor Jeremy Mynott.

My debts to several of the works cited in the bibliography will perhaps be obvious to those already acquainted with the subject of modal logic. In particular I should mention Lemmon and Scott's *Introduction to modal logic* and Segerberg's *Essay in classical modal logic*.

The contents of chapters 6 and 10 are largely adapted from my papers 'Imperatives', 'Conditional obligation', and 'Basic conditional logic', cited in the bibliography. I wish to thank Krister Segerberg, editor of *Theoria*, Sören Stenlund, editor of *Logical theory and semantic analysis*, Richmond Thomason, editor of the *Journal of philosophical logic*, and the D. Reidel Publishing Company for permission to use this material.

Steven Kuhn and Audrey McKinney read much of my manuscript at different stages of its development, and I am grateful to them for criticism and advice.

Krister Segerberg has been a mainstay of counsel and encouragement for many years. I have learned a great deal about modal logic from Segerberg, and I have benefited enormously from conversations with him in the course of writing this book.

Among many others who have contributed in various ways to this book I would like to thank Roy Benton, Paul Golden, Deborah Mayo, and Robert Pelcovits.

Finally, I owe an enduring debt to Dana Scott, who introduced me to modal logic, who taught me how to think about it, and whose conception of the subject fundamentally influenced my own.

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