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978-0-521-29442-3 - Mathematical Models for the Growth of Human Populations

J.H.Pollard

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J.H.POLLARD

MACQUARIE UNIVERSITY, SYDNEY, AUSTRALIA

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To my mother and my father

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Frontmatter

[More information](#)

Contents

	Preface	xi	
1	Introduction		<i>page</i> 1
2	The life table		3
2.1	<i>Introduction</i>		3
2.2	<i>Mortality computations</i>		4
2.3	<i>The force of mortality</i>		4
2.4	<i>Numerical evaluation of the force of mortality</i>		5
2.5	<i>Other life table functions</i>		6
2.6	<i>The graphs of certain life table functions</i>		7
2.7	<i>The Euler–Maclaurin expansion</i>		8
2.8	<i>The expectation of life</i>		9
2.9	<i>The uniform distribution of deaths</i>		10
2.10	<i>The concept of a stationary population</i>		11
2.11	<i>The multiple-decrement table*</i>		12
2.12	<i>The multiple-decrement table and its related single-decrement tables*</i>		14
2.13	<i>Examples</i>		16
2.14	<i>Exercises</i>		18
3	The deterministic population models of T. Malthus, A. J. Lotka, and F. R. Sharpe and A. J. Lotka		22
3.1	<i>Introduction</i>		22
3.2	<i>The continuous-time model of Sharpe and Lotka</i>		23
3.3	<i>The stable age distribution</i>		26
3.4	<i>Numerical values of r_0</i>		27
3.5	<i>The effects of the cumulants on r_0</i>		29
3.6	<i>Numerical values of A_0</i>		30
3.7	<i>Curve fitting the net maternity function</i>		31
3.8	<i>The momentum of population growth</i>		33
3.9	<i>An example</i>		34
3.10	<i>Exercises</i>		35

viii	<i>Contents</i>	
4	The deterministic theory of H. Bernardelli, P. H. Leslie and E. G. Lewis	page 37
4.1	<i>Introduction</i>	37
4.2	<i>The matrix method of P. H. Leslie</i>	38
4.3	<i>The theorem of Perron and Frobenius*</i>	39
4.4	<i>The latent roots of the Leslie matrix A</i>	42
4.5	<i>The asymptotic stable age distribution</i>	44
4.6	<i>The recurrence equation approach to the discrete-time model</i>	46
4.7	<i>The sensitivity of the intrinsic growth rate to changes in the age-specific birth and death rates</i>	48
4.8	<i>Some further investigations by P. H. Leslie</i>	49
4.9	<i>Some deterministic extensions</i>	50
4.10	<i>Weak ergodicity*</i>	51
4.11	<i>Competing populations*</i>	55
4.12	<i>An example</i>	57
4.13	<i>Exercises</i>	58
5	Simple birth and death processes	60
5.1	<i>Introduction</i>	60
5.2	<i>The Poisson process</i>	60
5.3	<i>The Yule process</i>	61
5.4	<i>A linear birth and death process</i>	62
5.5	<i>Kendall's birth, death and migration model</i>	65
5.6	<i>An example</i>	68
5.7	<i>Exercises</i>	69
6	The stochastic models of M. S. Bartlett and D. G. Kendall*	70
6.1	<i>Introduction</i>	70
6.2	<i>A two-type example due to Bartlett</i>	70
6.3	<i>Bartlett's more general model</i>	72
6.4	<i>The continuous-time model of D. G. Kendall</i>	75
6.5	<i>An alternative analysis of Kendall's model</i>	77
6.6	<i>Exercises</i>	80
7	The two-sex problem	82
7.1	<i>Introduction</i>	82
7.2	<i>The deterministic model of A. H. Pollard</i>	83
7.3	<i>The two-sex deterministic models of D. G. Kendall</i>	84
7.4	<i>Some simple marriage models</i>	86
7.5	<i>A simple model with a behavioural basis</i>	90

<i>Contents</i>	ix
7.6 <i>An early two-sex model due to L. A. Goodman</i>	<i>page</i> 91
7.7 <i>Age-specific dominance models</i>	93
7.8 <i>An example</i>	94
7.9 <i>Exercises</i>	95
8 <i>The extinction of surnames</i>	97
8.1 <i>Introduction</i>	97
8.2 <i>The Galton–Watson process</i>	99
8.3 <i>The calculations of A. J. Lotka</i>	102
8.4 <i>The multi-type process</i>	103
8.5 <i>Two examples</i>	109
8.6 <i>Exercises</i>	111
9 <i>The stochastic version of Leslie’s model</i>	112
9.1 <i>Introduction</i>	112
9.2 <i>Some preliminary results</i>	113
9.3 <i>The stochastic population model</i>	114
9.4 <i>The asymptotic behaviour of the means and variances</i>	116
9.5 <i>The effect of multiple births</i>	119
9.6 <i>An experiment using the stochastic model</i>	120
9.7 <i>Generalization to higher-order moments</i>	124
9.8 <i>Multi-type Galton–Watson processes with random branching probabilities</i>	129
9.9 <i>Derivation of the discrete-time equations from D. G. Kendall’s continuous-time theory*</i>	130
9.10 <i>Some examples</i>	133
9.11 <i>Exercises</i>	134
10 <i>Hierarchical population models and recruitment</i>	136
10.1 <i>Introduction</i>	136
10.2 <i>Gani-type models</i>	137
10.3 <i>Gani-type model – numerical example</i>	139
10.4 <i>The case of independent Poisson recruitment</i>	140
10.5 <i>The age structures of learned societies</i>	142
10.6 <i>Young-and-Almond-type models</i>	146
10.7 <i>Exercises</i>	146
11 <i>Conclusion</i>	148
<i>References</i>	150
<i>References according to chapter and section</i>	155

Cambridge University Press

978-0-521-29442-3 - Mathematical Models for the Growth of Human Populations

J.H.Pollard

Frontmatter

[More information](#)

x	<i>Contents</i>	
	Solutions to exercises	<i>page</i> 158
	Australian life table (males) 1961	175
	Author Index	179
	Subject Index	181

Preface

This book is an account of some of the mathematical models which have been proposed for studying the growth of human populations, and the emphasis is on stochastic models, i.e. systems that change in accordance with probabilistic laws. A few of these models are mentioned in other texts, but many of the more complex models appear only in original papers in diverse journals. The book began in the form of lecture notes prepared for graduate students in the Department of Statistics at the University of Chicago in the Spring Quarter of 1968, and experience gained at the University of Chicago and later at Macquarie University has suggested numerous modifications and additions to the original text.

The aim has been to make the book as self-contained as possible, and therefore the mathematical techniques used have been kept as elementary as possible. They are mostly standard undergraduate results of calculus, matrix algebra and basic statistics. There are a few instances in which rather more complicated methods are necessary, or less well-known techniques are employed, and then detailed explanations are given. One very important aspect of the subject has not been treated (except marginally): the estimation of numerical values for the life table. Specialist actuarial techniques are usually employed, and these numerical methods are described elsewhere.

The book is intended primarily as a reference work for demographic research workers, but it might be used as a textbook for a final-year undergraduate mathematics class or graduate demography students with strong backgrounds in mathematics.

Certain sections have been marked with asterisks: the material contained in them is not required for an understanding of later chapters, and they may be safely omitted, at least at a first reading. Exercises are given at the end of each chapter, and in solving these problems the student should learn a great deal more about the basic mathematical models involved. Solutions for these exercises are outlined on pages 158–174.

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Frontmatter

[More information](#)xii *Preface*

Fragments of the history of the subject are given in the main body of the text, and certain interesting historical references are quoted. References are given on pages 150–154.

I wish to thank Nathan Keyfitz for inviting me to the University of Chicago in 1967 and for the inspiration he has given me. I am indebted to Dr C. C. Heyde and Dr E. Seneta for telling me about Bienaymé's work in 1845 on the simple branching process and for allowing me to see the manuscript of their paper. Dr C. D. Cooper very kindly read an earlier version of chapter 4 and helped eliminate a major error in section 4.3; he also suggested the matrix in the final exercise of that chapter. Thanks are due also to Miss Jane Blaxland and Miss Helen Knight for typing several versions of the manuscript, and to Miss Betty Thorne for drawing the diagrams.

Finally, I should like to thank my wife for her patience during the many evenings when I was busy with this book.

Macquarie University, Sydney
January 1973

J.H.P.

NOTE TO 1975 IMPRESSION: Certain errors have come to light since this book was first published. These have now been corrected, and I wish to thank Graham Pollard for bringing some of these errors to my notice. I am also indebted to Professor H. O. Lancaster for drawing my attention to the work of Leonhard Euler (1707–83).

J.H.P.