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978-0-521-28882-8 - Mathematical Analysis: A Straightforward Approach, Second Edition

K. G. Binmore

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MATHEMATICAL ANALYSIS

A STRAIGHTFORWARD APPROACH

K. G. BINMORE

Professor of Mathematics

London School of Economics and Political Science

SECOND EDITION



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Frontmatter

[More information](#)

CONTENTS

	Preface to the first edition	ix
	Preface to the second edition	xi
1	Real numbers	1
1.1	Set notation	1
1.2	The set of real numbers	2
1.3	Arithmetic	3
1.4	Inequalities	3
1.9	Roots	6
1.10	Quadratic equations	6
1.13	Irrational numbers	9
1.14	Modulus	10
2	Continuum property	12
2.1	Achilles and the tortoise	12
2.2	The continuum property	13
2.6	Supremum and infimum	15
2.7	Maximum and minimum	15
2.9	Intervals	16
2.11	Manipulations with sup and inf	18
3	Natural numbers	20
3.1	Introduction	20
3.2	Archimedean property	20
3.7	Principle of induction	22
4	Convergent sequences	26
4.1	The bulldozers and the bee	26
4.2	Sequences	27
4.4	Definition of convergence	27
4.7	Criteria for convergence	30
4.15	Monotone sequences	33
4.21	Some simple properties of convergent sequences	37
4.26	Divergent sequences	38
5	Subsequences	41
5.1	Subsequences	41
5.8	Bolzano–Weierstrass theorem	47

Cambridge University Press

978-0-521-28882-8 - Mathematical Analysis: A Straightforward Approach, Second Edition

K. G. Binmore

Frontmatter

[More information](#)

<i>vi</i>	<i>Contents</i>	
5.12	Lim sup and lim inf	48
5.16	Cauchy sequences	50
6	Series	54
6.1	Definitions	54
6.4	Series of positive terms	55
6.7	Elementary properties of series	57
6.12	Series and Cauchy sequences	58
6.20	Absolute and conditional convergence	61
6.23	Manipulations with series	62
7	Functions	65
7.1	Notation	65
7.6	Polynomial and rational functions	67
7.9	Combining functions	69
7.11	Inverse functions	69
7.13	Bounded functions	71
8	Limits of functions	75
8.1	Limits from the left	75
8.2	Limits from the right	75
8.3	$f(x) \rightarrow l$ as $x \rightarrow \xi$	76
8.6	Continuity at a point	78
8.8	Connexion with convergent sequences	79
8.11	Properties of limits	80
8.16	Limits of composite functions	82
8.18	Divergence	83
9	Continuity	85
9.1	Continuity on an interval	85
9.7	Continuity property	87
10	Differentiation	92
10.1	Derivatives	92
10.2	Higher derivatives	93
10.4	More notation	94
10.5	Properties of differentiable functions	96
10.12	Composite functions	99
11	Mean value theorems	101
11.1	Local maxima and minima	101
11.3	Stationary points	102
11.5	Mean value theorem	103
11.9	Taylor's theorem	106

Cambridge University Press

978-0-521-28882-8 - Mathematical Analysis: A Straightforward Approach, Second Edition

K. G. Binmore

Frontmatter

[More information](#)

<i>Contents</i>		<i>vii</i>
12	Monotone functions	109
12.1	Definitions	109
12.3	Limits of monotone functions	109
12.6	Differentiable monotone functions	111
12.9	Inverse functions	111
12.11	Roots	113
12.13	Convex functions	115
13	Integration	120
13.1	Area	120
13.2	The integral	121
13.3	Some properties of the integral	122
13.9	Differentiation and integration	125
13.16	Riemann integral	128
13.19	More properties of the integral	130
13.27	Improper integrals	134
13.31	Euler–Maclaurin summation formula	136
14	Exponential and logarithm	138
14.1	Logarithm	138
14.4	Exponential	141
14.6	Powers	142
15	Power series	144
15.1	Interval of convergence	144
15.4	Taylor series	146
15.7	Continuity and differentiation	148
16	Trigonometric functions	151
16.1	Introduction	151
16.2	Sine and cosine	152
16.4	Periodicity	154
17	The gamma function	157
17.1	Introduction	157
17.2	Stirling's formula	157
17.4	The gamma function	158
17.6	Properties of the gamma function	160
18	Vectors	163
18.1	Introduction	163
18.2	Vectors	163
18.4	Length and angle in \mathbb{R}^n	164
18.8	Inequalities	167
18.10	Distance	168
18.12	Direction	169
18.13	Lines	169

Cambridge University Press

978-0-521-28882-8 - Mathematical Analysis: A Straightforward Approach, Second Edition

K. G. Binmore

Frontmatter

[More information](#)

<i>viii</i>	<i>Contents</i>	
18.15	Hyperplanes	171
18.18	Flats	173
18.21	Vector functions	174
18.22	Linear and affine functions	175
18.26	Convergence of sequences in \mathbb{R}^n	181
18.31	Convergence of functions	183
18.33	Continuity at a point	184
18.35	Properties of limits	185
18.40	Limits along a path	186
19	Vector derivatives	190
19.1	Directional derivatives	190
19.3	Partial derivatives	192
19.5	Notation	194
19.7	Local maxima and minima	196
19.12	Differentiable functions	199
19.14	Derivatives	203
19.19	Gradient	210
19.21	Manipulation of derivatives	211
19.23	Chain rule	213
19.27	Stationary points	217
19.31	Second derivatives	219
19.34	Mean value theorems	221
19.37	Eigenvalues	223
19.38	Principal minors	225
19.40	Classification of stationary points	227
19.48	Differentials	231
20	Appendix	237
	This contains the proofs of 'propositions' left unproved in the main body of the text.	
	Solutions to exercises	251
	Further problems	345
	Suggested further reading	355
	Notation	357
	Index	359

PREFACE TO THE FIRST EDITION

This book is intended as an easy and unfussy introduction to mathematical analysis. Little formal reliance is made on the reader's previous mathematical background, but those with no training at all in the elementary techniques of calculus would do better to turn to some other book.

An effort has been made to lay bare the bones of the theory by eliminating as much unnecessary detail as is feasible. To achieve this end and to ensure that all results can be readily illustrated with concrete examples, the book deals only with 'bread and butter' analysis on the real line, the temptation to discuss generalisations in more abstract spaces having been reluctantly suppressed. However, the need to prepare the way for these generalisations has been kept well in mind.

It is vital to adopt a systematic approach when studying mathematical analysis. In particular, one should always be aware at any stage of what may be assumed and what has to be proved. Otherwise confusion is inevitable. For this reason, the early chapters go rather slowly and contain a considerable amount of material with which many readers may already be familiar. To neglect these chapters would, however, be unwise.

The exercises should be regarded as an integral part of the book. There is a great deal more to be learned from attempting the exercises than can be obtained from a passive reading of the text. This is particularly the case when, as may frequently happen, the attempt to solve a problem is unsuccessful and it is necessary to turn to the solutions provided at the end of the book.

To help those with insufficient time at their disposal to attempt all the exercises, the less vital exercises have been marked with the symbol †. (The same notation has been used to mark one or two passages in the text which can be omitted without great loss at a first reading.) The symbol * has been used to mark exercises which are more demanding than most but which are well worth attempting.

The final few chapters contain very little theory compared with the number of exercises set. These exercises are intended to illustrate the power of the techniques introduced earlier in the book and to provide the opportunity of some revision of these ideas.

This book arises from a course of lectures in analysis which is given at the London School of Economics. The students who attend this course are mostly not specialist mathematicians and there is little uniformity in their previous

Cambridge University Press

978-0-521-28882-8 - Mathematical Analysis: A Straightforward Approach, Second Edition

K. G. Binmore

Frontmatter

[More information](#)*x Preface to the first edition*

mathematical training. They are, however, quite well-motivated. The course is a 'one unit' course of approximately forty lectures supplemented by twenty informal problem classes. I have found it possible to cover the material of this book in some thirty lectures. Time is then left for some discussion of point set topology in simple spaces. The content of the book provides an ample source of examples for this purpose while the more general theorems serve as reinforcement for the theorems of the text.

Other teachers may prefer to go through the material of the book at a more leisurely pace or else to move on to a different topic. An obvious candidate for further discussion is the algebraic foundation of the real number system and the proof of the Continuum Property. Other alternatives are partial differentiation, the complex number system or even Lebesgue measure on the line.

I would like to express my gratitude to Elizabeth Boardman and Richard Holmes for reading the text for me so carefully. My thanks are also due to 'Buffy' Fennelly for her patience and accuracy in preparing the typescript. Finally, I would like to mention M.C. Austin and H. Kestelman from whom I learned so much of what I know.

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Frontmatter

[More information](#)

PREFACE TO THE SECOND EDITION

It is a pleasure to write a preface for the second edition of *Mathematical Analysis: A Straightforward Approach*. The first edition was well-received and I have therefore thought it wise to leave its text substantially unaltered except for one or two minor points of clarification and the correction of misprints. The major change is the addition of two long chapters on analysis in vector spaces for which there has been a considerable demand. These get as far as the idea of a derivative as a matrix and the use of the second order derivative of a real-valued function in classifying stationary points. More advanced material than this would seem to me better delayed until after the basic topological notions have been mastered. As far as the material covered is concerned, it does not involve the proof of many theorems and the necessary proofs involve no new analytic ideas. However, the material does require a certain facility with algebraic and geometric ideas and students with only a very limited knowledge of linear algebra may find it heavy going in spite of the fact that some discussion of the necessary concepts from linear algebra is included where appropriate. Another innovation is the inclusion of a collection of further problems for which the solutions are not given. I am grateful to John Erdős for some of these as well as other helpful suggestions. Teachers using this book as part of a taught course may find these problems helpful in setting work but I hope that they will not distract attention from the importance of working carefully through the exercises given in the main body of the text.

Finally, I would like to express my appreciation to those who have commented favourably on the first edition and to Mimi Bell for her patient help in preparing the typescript for the second edition.

October 1981

K.G.B.