The design of experiments

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STATISTICAL PRINCIPLES FOR PRACTICAL APPLICATIONS

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CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011–4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1988
First paperback edition (with corrections) 1990
Reprinted 1991, 1992, 1994

British Library cataloguing in publication data
Mead, R.
The design of experiments: statistical
principles for practical applications.
1. Experimental design
I. Title
001.4'34'028 QA279

Library of Congress cataloguing in publication data
Mead, R. (Roger)
The design of experiments
Bibliography
Includes index.
1. Experimental design. 2. Mathematical statistics.
I. Title
QA279.M388 1988 001.4'34 87-24236

ISBN 0 521 24512 5 hardback
ISBN 0 521 28762 6 paperback

Transferred to digital printing 2000

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PREFACE

My aim in this book is to explain the fundamental statistical concepts required for designing efficient experiments to answer real questions. The book has grown out of 25 years experience of designing experiments for research scientists, and of teaching the concepts of statistical design both to statisticians and to experimenters. This experience has convinced me that the whole subject of statistical design needs to be reassessed in the context of modern statistical computing facilities. The development of statistical philosophy about the design of experiments has been dominated during the last 30 years by mathematical theory. The influence of the availability of vastly improved computing facilities on teaching, textbooks, and, most crucially, practical experimentation appears to have been slight.

The existence of statistical programs capable of analysing the results from any experimental design does not imply any changes in the main statistical concepts of design. However these concepts have become restricted by the earlier need to develop mathematical theory for design in such a way that the results from the designs can be analysed without recourse to computers. The fundamental concepts now require re-examination and re-interpretation outside the limits implied by classical mathematical theory so that the full range of design possibilities may be considered. The result of the revolution in computing facilities is that experimental design should become a much wider and more exciting subject. I hope that this book will display that breadth and excitement.

The development of this book has been particularly motivated by teaching postgraduate students specialising in statistics. However the intention of the book encompasses a wider audience. Understanding the fundamental concepts of design is essential for all research scientists involved in programmes of experimental work. In addition to this general need for an understanding of the philosophy of experimental design there

are particular aspects of design, such as repeated measurements (Chapter 14) or levels of replication (Chapter 6) which are relevant to virtually all research disciplines in which experimental work is required. Because of the concentration on basic concepts and their implications the book could be used for courses for final-year undergraduates provided such courses allow sufficient time for the concepts to be thoroughly discussed.

I have tried in writing this book to concentrate on the ideas of design rather than those of analysis, on the statistical concepts rather than the mathematical theory, and on practically useful experiments rather than classes of possible experimental designs. Obviously it is necessary to know how to analyse data from experiments and the philosophy and methods of analysis are discussed in the introductory first part of the book. Also of course, examples of analysis punctuate many later sections. However, after the first part of the book it is assumed that the analysis of experimental data is not difficult when modern computing facilities are used. Consequently ideas of analysis are introduced only when they illumine or motivate the design concepts.

The formal language of statistics is mathematical. It is not possible to discuss design without some mathematically complex formulation of models and ideas. Some of the mathematical language used in the book requires a sound mathematical background beyond school level. However in all parts of the book it is the statistical concepts which are important, and the structure of the book hopefully allows the less-mathematical reader to bypass the more complex mathematical details. Throughout the book the development of concepts relies on many examples. I hope that readers will consider the detailed arguments of these examples. By trying to solve the problems which underly the examples before reading through the explanation of the solutions, I believe that readers will start to develop the intuitive understanding of design concepts which is essential to good design. For the mathematically sophisticated reader the mathematical details provide additional support for the statistical concepts.

Most importantly, the book is intended to show how practical problems of designing real experiments should be solved. To stimulate this practical emphasis real examples of design problems are described at the beginning of each chapter from Chapter 6 onwards (except Chapter 9). The final chapter of the book attempts an overall view of the problem-solving aspects of design. The bias of the areas of application discussed in this book inevitably reflects my personal experience. Much of my experience has been concerned with agricultural experimentation and to write with anything like a comprehensive knowledge about the practical application of design concepts I believe that a concentration on agricultural appli-

cations is inevitable. However all the examples are intended to illustrate particular forms of problem that will be relevant in many other fields of application. I hope that statisticians and research scientists in a wide range of experimental disciplies will be able to interpret and adapt the concepts discussed in the book to their own requirements through the use of analogy when the examples discussed are not directly relevant to their discipline.

The book is divided into an overture and two main subjects with a final coda to bring together all the previous material. Chapters 1 to 5 constitute the overture, providing a general introduction and the basic theory necessary for analysis of experimental data. Chapters 1, 2 and 3 should be familiar to readers who have taken an elementary course in experimental design. Alternatively, for those readers without any previous training in design, these three chapters provide an introductory presentation of the two most important ideas of design, blocking and factorial structure. Chapter 4 is the mathematically heavy chapter, providing the necessary theory for general linear models and the analysis of data from designed experiments, with an initial explanation of the important results at a rather simpler level. Chapter 5 ventures briefly, and rashly, into an inevitably prejudiced view of computing needs and implications.

The first main subject is unit variation and control. The fundamental concepts of replication, blocking (with either one or two systems of control) and randomisation are each examined separately. My aim is to distinguish the purposes and practical relevance of each concept and to eliminate the confusion about these concepts which seems to be common in the minds of users of experimental designs. The other two chapters in this subject, on covariance (for control) and on assumptions (to express variation realistically) are biased towards analysis rather than design but their implications for design and for the choice of measurement variables are important.

The second main subject is treatment questions and structure. Chapter 12 presents a broad view of the need for statisticians to be involved in all stages of discussion about the choice of treatments and the interpretation of results. The classical ideas of factorial structure and single and fractional replicates are presented in Chapter 13. Important consequences of particular practical requirements for factorial structure are described in Chapters 14 (split levels of information) and 15 (avoiding confounding). Some necessary mathematical theory for confounding is included in Chapter 16. The choice of experimental treatments for the investigation of the response to quantitative factors is discussed in Chapters 17 and 18.

Finally in the coda, Chapter 0 seeks to draw the concepts of the two

main subjects together to provide guidance on choosing experimental designs to satisfy particular practical requirements. A book on practical experimental design should start with the approach of Chapter 0 but this chapter requires knowledge from the previous chapters before it can be read and understood. Hence the number and position of this chapter.

I owe a considerable debt to many consultees and collaborators both for the stimulus to consider why the problems they presented should be covered by a book on design and also for the many examples they have provided. There are too many for me to thank them individually here for their stimulating requests and they therefore remain anonymous (some should prefer it that way, and others are too far into the mists of time for anything else). I have also drawn exercises at the end of the chapters from Chapter 6 onwards from many examination papers.

I have also benefitted from many discussions with colleagues at Reading and wish to thank Richard Coe, Robert Curnow, Derek Pike and Roger Stern, without whom this book would have been a more stunted growth. I am particularly grateful to Richard Jarrett for a most stimulating dinner conversation in Melborne, from which Chapter 0 was born.

Finally I record my gratitude to those who have made the production of this book possible. Clive Bowman and Marilyn Allum manipulated GLIM with great skill to provide many analyses and information about precision for possible designs. Audrey Wakefield has worked devotedly through thousands of pages of manuscript, second thoughts, third thoughts, total reorganisations..., and has endured the slings and arrows of an early generation word-processor. Rosemary Stern has used her art to convert ideas into figures. And David Tranah and Martin Gilchrist at Cambridge University Press have been most patient with a tardy author.