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Symmetric designs: an algebraic approach

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To my teachers, Peter J. Cameron
and John C. Moore
and to my wife, Lori Ann.

Δοσ που στω ...

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PREFACE

Researchers studying the theory of error-correcting codes have discovered, in recent years, that finite geometries and designs can provide the basis for excellent communications schemes. The basic idea is to take the linear span (over some finite field) of the rows of the incidence matrix of such a structure as the allowable messages. Mariner 9, for example, transmitted data to Earth by using a code derived from the structure of the hyperplanes in a five-dimensional vector space over F_2, the field with two elements.

The purpose of this monograph is to allow coding theory to repay some of its debt to the combinatorial theory of designs. Specifically, I have tried to show herein how the objects introduced by coding theorists can offer great insight into the study of symmetric designs.

The vector spaces and modules (over appropriate rings) generated by the incidence matrices of symmetric designs provide a natural setting for invoking much algebraic machinery--most notably, the theory of group representations--which has hitherto not found much application in this combinatorial subject. In doing so, they provide a point of view which unifies a number of diverse results as well as makes possible many new theorems. My own investigation into this subject is surely not definitive, and if anyone is stimulated to further develop this point of view, I will have accomplished something.

Two goals have informed my choice of organization. First, since my object is to expose a particular approach to the study of symmetric designs, I have chosen to develop the subject from scratch. This also seemed appropriate because it should make the topic accessible to readers who only know a little combinatorics, and because no text yet exists squarely devoted to the subject of symmetric designs.

Second, it is my goal to advocate the increased use of powerful algebraic techniques in combinatorics. I realize that some of the combinatorialists who will read this monograph may not be familiar or comfortable with a number of the algebraic topics I shall employ. Rather than set prerequisites which might dissuade potential readers, I have included six appendices and two sections of supplementary problems which introduce and develop the techniques I shall require. These treatments are intended merely to be superficial and expedient --certainly not exhaustive. Rather, I hope the reader may choose to learn more about some of the topics by reading serious treatments of them later.

The only prerequisites are a first course in algebra (groups, rings, fields, modules); a smattering of number theory (through quadratic reciprocity, although an acquaintance with algebraic number theory would be helpful in a few places); and a knowledge of the basic counting principles and questions studied in combinatorics.

I have attempted to make this monograph useful to advanced undergraduates or graduate students as either a text or for self-study--as well as to the researcher active in the study of designs. Toward this end, I have included 126 problems to be solved, at the end of the chapters.

The text proceeds as follows. Chapter 1 contains an introduction to the basic notions about symmetric designs and provides a stock of examples upon which we draw throughout the text. Most of the material in this chapter is well-known to specialists, although a set of supplementary exercises developing an application of algebraic geometry to symmetric designs is new. Chapter 2 begins with the question of existence criteria for symmetric designs and proves the celebrated Bruck-Ryser-Chowla Theorem in the (more-or-less) standard way. Then we introduce the modules and vector spaces which are the main tools of this monograph, study their properties and use them to reinterpret the Bruck-Ryser-Chowla Theorem. Chapter 3 studies automorphisms of symmetric designs, applying group representation theory to obtain theorems which supersede the previously known results due to Hughes. The methods are most effective in the case that a regular permutation group acts on the symmetric design in question and so in Chapter 4 we study the difference sets which arise in this case. Chapter 5 continues this study by presenting a number of multiplier theorems (including some new ones) for difference sets. An important aspect of Chapters 4 and 5 is that certain results previously thought to be unique to abelian difference sets are in fact special cases of more general results for symmetric designs. Finally, Chapter 6 raises some open questions concerning difference sets and presents tables of data concerning existence of particular difference sets. These tables are the product of my own hand calculations and I would be most grateful to anyone who can fill in any of the question marks that remain (or correct any errors I have made). The six appendices dealing with algebraic topics follow.

I warmly acknowledge my many debts. First and foremost, I owe much to Peter Cameron, my thesis supervisor, for all he has taught me--mathematically and otherwise--while advising me on the work which has led to this book; I hope he realizes how important his influence has been. I am also grateful to Peter Neumann for his many helpful conversations and for allowing me to wheedle him into teaching a wonderfully useful course on permutation modules in 1980 at Oxford.

I thank Jack van Lint for inviting me to the Technical University of Eindhoven, Netherlands, to give five weeks of lectures from an earlier version of this manuscript. I profited greatly from my contact with the members of his algebraic combinatorics seminar; in particular, I thank J. J. Seidel, H. Wilbrink, A. Brouwer, and A. Cohen. I also spent one of the most pleasant days of my mathematical career in Amsterdam with Hendrik Lenstra, who generously helped me sort out a number of questions. I also gratefully acknowledge the financial support of the Rhodes Trust, Wolfson College (Oxford), The Mathematical Institute (Oxford), and Harvard University.

Peter Cameron, Jack van Lint and Ed Assmus all provided very useful remarks on the manuscript. Susan Landau and Neil Immerman graciously agreed to assist in proofreading, for which I thank them. I, of course, am fully responsible for all errors--mathematical, calculational, and typographical--which remain.

To Pat Giersz, my secretary, who did an excellent job typing, retyping, and revising this book under trying circumstances, I offer my deep gratitude--although I suspect she would prefer a promise that I will not undertake another project like this for at least twelve months.

Finally, there is my wife, Lori. I do not know where to begin to thank her for the support, interest, and encouragement which made this project a reality and for her ability, willingness, and patience in discussing subject matter at length, despite having no training in mathematics. I count myself a lucky man, indeed.