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E. J. N. Looijenga

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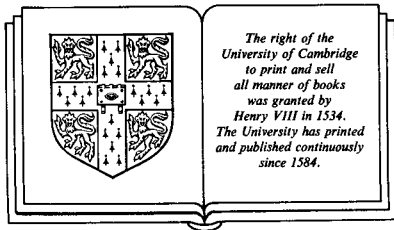
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INTRODUCTION

In the spring term of 1980 I gave a course on singularities at Yale University (while supported by NSF grant MCS 7905018), which provided the basis of a set of notes prepared for the first two years of the Singularity Intercity Seminar (1980 - 1982, at Leiden, Nijmegen and Utrecht, jointly run with Dirk Siersma and Joseph Steenbrink). These notes developed into the present book. As a consequence, aim and prerequisites of the seminar and this book are almost identical.

The purpose of the seminar was to introduce its participants to isolated singularities of complex spaces with particular emphasis on complete intersection singularities. When we started we felt that no suitable account was available on which our seminar could be based, so it was decided that I should supply notes, to be used by both the lecturers (in preparing their talks) and the audience. This was quite a purifying process: many errors and inaccuracies of the first draft were thus detected (and often corrected).

The prerequisites consisted of some algebraic and analytic geometry (roughly covering the contents of the books of Mumford (1976) and Narasimhan (1966)), some algebraic topology (as in Spanier (1966) and Godement (1958)) and some facts concerning Stein spaces. Given this background, my goal was to prove every assertion in the text. This has been achieved except for the coherence theorem (8.7) and some assertions

in the descriptive chapter 1. An exception should also be made for the paragraphs marked with an asterisk (*): they generally give useful information, which however is not indispensable for what follows and so may be skipped. Perhaps the whole first chapter could have been marked with an asterisk. It gives interesting examples of isolated singularities (or of constructions thereof) with the purpose to indicate the position of complete intersection singularities among them and to describe material to which the theory is going to apply. It is mainly for the latter reason that this chapter should not be entirely skipped.

As each chapter has its own introduction, I shall not review the chapters separately, nor the whole book. I believe that the first seven chapters (with the exception of § 5.C) can be used as a basis for a course on the subject, assuming the audience has approximately the background described above. The contents of § 5.C and the last two chapters are somewhat more advanced and in addition, chapter 9 is of a more specialized nature. Some results or proofs may be new (at least do not appear in this form in the literature), examples are Ch. 1, p. 18ff, (4.7), (4.11), § 5.C, (6.13), (7.14), § 9.A, § 9.C. The references at the end should be regarded as a list of sources I consulted and not as a bibliography which aims to be complete in any respect.

Acknowledgements. Although all sources I used are cited, I want to single out some papers which were particularly useful to me: Lamotke (1975) for Ch. 3, Teissier (1976) for Ch. 4, Lê (1973, 1978) for Ch. 5 and Greuel (1975, 1980) for Ch. 8 and 9. As already mentioned, the book benefitted from criticism of the lecturers in our seminar. I mention in particular C. Cox, W. Janssen, P. Lemmens, P. Lorist, F. Menting,

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August 1983

E. Looijenga