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London Mathematical Society Lecture Note Series : 54

Markov Processes and Related Problems of Analysis

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CAMBRIDGE UNIVERSITY PRESS

Cambridge

London New York New Rochelle

Melbourne Sydney

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521285124

First published 1982
Re-issued in this digitally printed version 2008

A catalogue record for this publication is available from the British Library

Library of Congress Catalogue Card Number: 81–38438

ISBN 978-0-521-28512-4 paperback

CONTENTS

Preface

Most of the papers compiled in this volume have been published in Uspekhi Matematicheskikh Nauk and translated into English in the Russian Mathematical Surveys. The core consists of the series [IV], [V], [VI], [VII] presenting a new approach to Markov processes (especially to the Martin boundary theory and the theory of duality) with the following distinctive features:

1. The general non-homogeneous theory precedes the homogeneous one. This is natural because non-homogeneous Markov processes are invariant with respect to all monotone transformations of time scale – a property which is destroyed in the homogeneous case by the introduction of an additional structure: a one-parameter semi-group of shifts. In homogeneous theory, the probabilistic picture is often obscured by the technique of Laplace transforms.

2. All the theory is invariant with respect to time reversion. We consider processes with random birth and death times and we use on equal terms the forward and backward transition probabilities, i.e., the conditional probability distributions of the future after t and of the past before t given the state at time t. (This is an alternative to introducing a pair of processes in duality defined on different sample spaces.)

3. The regularity properties of a process are formulated not in topological terms but in terms of behaviour of certain real-valued functions along almost all paths. Specifying a countable family of the base functions, we introduce a topology in the state space such that almost all paths have certain continuity properties. However this can be done in many different ways with different exceptional sets of paths. It is reminiscent of the situation with coordinate systems: there exist many of them and we have no reason to prefer any special one.

Two recent papers [VII] and [VIII] are closely related to the main series.

An earlier article [I] (its title is used as the title of the volume) presents the state of the theory of Markov processes in 1959. At this time the theory was in the process of extensive development and Markov processes attracted researchers around the world. The article is a report on the work done by a group of young mathematicians at Moscow University (almost all of them were in their twenties). A number of open problems and prospective directions have been mentioned in the article. Two of them: additive functionals of Markov processes and applications of Ito's stochastic differential equations to partial differential equations – became a major area of research in subsequent years. Three years later the progress was reported in monograph [1].

The boundary theory of Markov processes is one of the principal subjects of the volume. In [II] a boundary value problem with a directional derivative for the Laplace equation is studied. At that time the general theory had not been sufficiently developed and the first sections of [II] are devoted to adjustment of Martin's method.

A general boundary theory is presented in [IV] and [V]. It is based on a theorem concerning the decomposition of certain classes of measures into extreme elements. An improved version of this theorem with applications to a number of other problems is contained in [VIII]. The key role is played by a special type of sufficient statistics. Under minimal assumptions on a transition function, the corresponding entrance and exit spaces are evaluated in [IX] using a combination of the boundary theory and the ergodic theory.

The relation of the general boundary theory to Hunt's boundary theory for Markov chains can be easily seen in an earlier paper [III]. The main difference is in the way a Markov chain (X_t, P) with given transition probabilities is associated with an excessive measure μ. In Hunt's theory $\mu(x)$ is the expected number of hittings x by X_t during the life interval $[\alpha, \beta]$. In our approach $\mu(x) = P\{\alpha \leqslant t, X_t = x, t \leqslant \beta\}$. This modification makes possible the generalization presented in [IV] and [V].

Papers [VI] and [VII] are devoted to the problem of constructing Markov processes whose paths have certain regularity properties. The class of regular processes investigated in [VI] is close to the class of right processes introduced by Meyer and studied by Getoor [4]. The theory of Markov representations of stochastic systems developed in [VII] presents an alternative to the classical theory of duality due to Hunt, Kunita, Watanabe, Getoor, Sharp and others. The relation between both theories is discussed in [2]. Additive functionals of stochastic systems have been studied in [3]. Interesting results in spirit of [VII] have been obtained by Kuznecov [5], [6], [7], [8], and [9] and Mitro [10], [11].

For this edition the author has revised the entire text of the English translations. A few slips in the originals (some of them noticed by Kuznecov) have also been corrected.

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Roman figures refer to the articles collected in this volume (see the table of contents).