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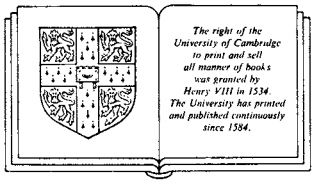
Spectral Theory of Linear Differential Operators and Comparison Algebras

H. O. CORDES
University of California, Berkeley



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PREFACE

The main purpose of this volume is to introduce the reader to the concept of comparison algebra, defined as a type of C^*-algebra of singular integral operators, generally on a noncompact manifold, generated by an elliptic second order differential expression, and certain classes of multipliers and 'Riesz-operators' As for singular integral operators on $\mathbb{R}^n$ or on a compact manifold the Fredholm properties of operators in such an algebra are governed by a symbol homomorphism. However, for noncompact manifolds the symbol is of special interest at infinity. In particular the structure of the symbol space over infinity is of interest, and the fact, that the symbol no longer needs to be complex-valued there.

The first attempts of the author to make a systematic presentation of this material happened at Berkeley (1966) and at Lund (1970/71). Especially the second lecture exists in form of (somewhat ragged) notes [CS] . The cases of the Laplace comparison algebra of R^n and the half-space were presented in [C_1] .

In the course of laying out theory of comparison algebras we had to develop in details spectral theory of differential operators, as well as many of the basic properties of elliptic second order differential operators. This was done in the first four chapters. Comparison algebras (in L^2-spaces and L^2-Sobolev spaces) are discussed in chapters V to IX . Finally, in chapter X we recall the basic facts of theory of Fredholm operators, partly without proofs.

The material has been with the author for more than 20 years and has been subject of innumerable discussions with students and

associates. Accordingly it is almost impossible to recall in detail the origin of the various concepts introduced. Especially we are indebted to E. Herman, M. Breuer, E. Luft, M. Taylor, R. McOwen, A. Erkip, D.Williams, H. Sohrab, in chronological, not alphabetical order.

We are indebted to S.H.Doong, S.Melo, R.Rainsberger, M.Arsenovic for help with proof reading. This volume originally was planned under the title 'techniques of pseudodifferential operators', but then split into two parts, with the second yet to appear. We are grateful to the publisher, Cambridge University Press, for cooperation and patience in waiting for the manuscript.

Berkeley, September 1986 Heinz O. Cordes

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