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Circuit Double Cover of Graphs

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To

To my mentors, colleagues, and students –
who make research exciting

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Cun-Quan Zhang
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Foreword

When I use the term *multigraph decomposition*, I mean a partition of the edge set. In particular, a *cycle decomposition* of a multigraph is a partition of the edge set into cycles, where I am using *cycle* to indicate a connected subgraph in which each vertex has valency 2.

There is a short list of cycle decomposition problems that I view as important problems. At the top of my list is the so-called *cycle double cover conjecture* which is the underlying motivation for this book. My reasons for ranking it at the top are discussed next.

If a conjecture has been largely ignored, then longevity essentially is irrelevant, but when a conjecture has been subjected to considerable research, then longevity plays a significant role in its importance. The cycle double cover conjecture has been with us for more than thirty years and has received considerable attention including three special workshops devoted to just this single conjecture. Thus, just in terms of longevity the cycle double cover conjecture acquires importance.

There is a deep, but not well understood, connection with the structure of graphs for if a graph X contains no Petersen minor, then a vast generalization of the cycle double conjecture is true. Trying to understand what is going on in this realm adds considerably to the allure of the cycle double cover conjecture.

Another strong attraction of the conjecture is the connections with other subareas of graph theory. These include topological graph theory, graph coloring, and flows in graphs.

This book provides a thorough exploration of the general conjecture, the approaches that have been developed, connections with the subareas mentioned above, and is presented in a highly readable form. Anyone interested in the conjecture will welcome this addition to the mathematical literature.

I wish to close the foreword with a few words about the author, Cun-Quan Zhang, known to his friends as CQ. His story is a remarkable testament to perseverance.

He was born and raised in the People's Republic of China. When the cultural revolution began in May 1966, CQ was in the seventh grade. His school, like most, closed at that time and many schoolmates became Red Guards. Due to some kind of family background problem, he was considered unqualified to join them.

He was sent to a mountain village in 1969 in Guizhou Province in western China. One year later he started to pick up all the lost education of middle school and high school. He did this with no teacher and no help. His only possibility was to teach himself.

There was nothing to do in the rice fields during Winter so that was the best time to read books. During Spring, Summer, and Fall, when there was work to be completed in the fields, he was able to read only by candlelight.

Surprisingly, textbooks were not a problem as they were available in used-book stores in Shanghai (science was considered useless at that time). The cost of a set of calculus books was no more than a pack of cigarettes.

After he finished learning high school material, he started to teach himself basic university courses in physics, mathematics, chemistry, and electrical engineering. Mathematics was the subject he loved the most.

Universities opened in 1972, but only for children from revolutionary families. He did not qualify. Also, entrance exams were not required as government selection and family background were the keys for admission.

His real mathematics career started after the cultural revolution. Chairman Mao died in 1976 and two years later university re-opened for the general public. The only requirement for entrance was a federal entrance exam.

He took an entrance exam for a graduate program in 1978, and was admitted into a master's program in mathematics in 1978 (Qufu Normal College in the hometown of Confucius). His master's thesis adviser was Professor Yongjin Zhu in the Chinese Academy of Science. He was hired by the Chinese Academy of Science as a junior faculty in 1981. Six months later, in the Fall of 1982, he came to Simon Fraser University to begin a Ph.D. under my supervision.

He was the first student from mainland China to undertake graduate studies in mathematics at Simon Fraser University. He set a very high standard for the students that followed him and was a joy to supervise. Upon completion of his Ph.D., he took a position at West Virginia University where he remains to this day.

I have watched his career with great interest. He has constantly maintained excellent standards in his research, teaching and graduate supervision. I am happy to have known him and to be able call him a friend for almost thirty years. Well done, CQ!

Brian Alspach
Callaghan, Australia

Foreword

It has been more than thirty years since I first encountered the Circuit Double Cover Conjecture. A colleague had approached me in the hallway, with what he then referred to as “a nice little problem.” Indeed, even back then, the problem had already been floating here and there for quite some time. Communication, however, was not remotely what it is today and new ideas spread around erratically and at a very slow pace.

It did not seem difficult. I thought, at first, that he meant it to be an exercise for our Graph Theory course, and was somewhat embarrassed, as I was not able to solve it right away. “Every edge doubled,” I was thinking out loud, “that makes an Eulerian graph. ... bridgeless, so there should be a simple way to construct a circuit partition, that avoids both copies of an edge on the same circuit... Well, I will think about it.”

Three decades and hundreds of related publications later, and I still think about it, and so do many others. Indeed, a fascinating “nice little problem.”

No serious mathematical question, solved or unsolved, is as simple to state and as easy to understand. No background is required; nothing essential about graphs; not even basic arithmetic. Take the intuitive concept of a line joining two points, the idea of following such lines back to the starting point to form a circuit, the ability to count “one, two” and voilà, you have the Circuit Double Cover Conjecture.

Yet, at least until the conjecture is settled, we cannot be sure that any other problem is actually harder to solve.

That said, the CDC conjecture appears to lack the fame and glamour associated with some other, celebrated and publicly praised mathematical problems. When searching the literature and the electronic space for lists of open mathematical questions, the CDC conjecture scarcely comes up. It never does among “The most famous problems,” (or “most important,” or “most elegant”). It gets somewhat better when the search is restricted to Graph Theory, but even then, if found, the CDC conjecture is located far down on such lists. Whatever reasons are behind that situation, C.-Q. Zhang’s book in hand can be a step toward changing it.

In terms of mathematical genealogy, the CDC conjecture descends from a dignified noble dynasty, the family of “Circuit Cover” problems. Its oldest ancestor is Leonard Euler’s work, dated 1736, on the Seven Bridges of Königsberg, which many consider as the very start of graph theory. Family resemblance may be superficial, yet definitely apparent – Euler’s work can well be titled “The Theory of Circuit SINGLE Covers.”

Another prominent grandparent is the Four-Color Theorem, formerly, the long standing Four-Color Conjecture. Common family features here are more deeply hidden and harder to spot. A four-coloring of a planar map is equivalent to a circuit double cover, which can be double covered by three even subgraphs. That simple (but not straightforward) observation led W. T. Tutte to develop his theory of nowhere zero flows, which gave birth to some additional noteworthy members of the family, among them the 5-nowhere zero flow and the 3-nowhere zero flow conjectures, now standing open for over half a century. The assertion of the CDC conjecture can also be equivalently formulated in terms of nowhere zero flows of a certain kind.

Once the family circle is allowed to grow wider, it includes the entire theory of graph-coloring. To see that, one should adopt a matroid theoretical approach, by which nowhere zero flows and graph proper vertex colorings become one. When stated within matroid theory, the CDC conjecture (for graphs) is equivalent to its generalization to regular matroids. However, with the wider community of graph theory researchers, students and enthusiasts in mind, the author deliberately chose to avoid matroid theoretical concepts and terminology in his book.

The branch of that family, rooted in the seminal work of Tutte (matroids excluded), was the subject of C.-Q. Zhang's first book: *Integer Flows and Cycle Covers of Graphs*. In his current book, the author zooms in to focus on the rich body of results, methods and questions, directly and indirectly related to the CDC conjecture.

C.-Q. Zhang's fascination with nowhere zero flow theory, circuit covers and the CDC conjecture, alongside his expertise in the areas led to his becoming a center of activity in the field, as well as the holder of a knowledge data base and a living communication hub for those who share his fascination. Most of the related new questions, new results and new ideas are addressed first to him, for verification, and comparison against the vast, ever growing and updating body of knowledge that he possesses.

In his book, Zhang draws a comprehensive panoramic image of the continuing quest for a solution to the CDC conjecture. Hundreds of related research results have spread over time across a wide variety of journals, conference proceedings, lecture notes, internal reports and private correspondence. The most essential of these results (more than a few due to Zhang himself), not only are cited in the book, but are virtually rewritten (some meanwhile improved) and the fabric of connections among them revealed. Zhang has established a uniform framework in which most of the work done so far, as well as potential directions for future work are described and understood in a clear and systematic manner.

Among the various methods and lines of action, which may lead to a solution, an attentive reader can identify the author's own plan and vision. It appears to be a rather long and involved chain of arguments, that he mostly developed himself, with an affirmative answer to the CDC conjecture at the end. Although more than a sketchy schema, it is very clearly less than a proof (and certainly not claimed to be one). The mostly paved looking path is fragmented by obstacles which, while few in number, are significant. Can all these obstacles be bridged over to complete a valid proof?

One way or another, most experts regrettably agree that the days of reasonably long and elegant solutions to long standing open problems are over. A solution to the CDC conjecture is expected to be nothing but a long sequence of rather complex arguments, rich with case analysis and technical details and hard to follow and verify. This does not at all exclude the vital need for brilliant novel methods and sparkling new ideas. An intensive coordinated team effort may be the right tool to apply here. Some activity is indeed conducted with that prospect in mind, for example, the two weeks long workshop, solely devoted to the CDC conjecture, which was held in Vancouver, British Columbia, in August 2007.

When will the CDC conjecture finally be settled? Within one month? One year? A decade? Another century? Or was it already solved last week?

My Ph.D. advisor, the late professor Haim Hanani, published his own Ph.D. dissertation back in 1938. The thesis dealt with some aspects of the Four-Color Conjecture. Thirty eight years later, during my studies under his guidance, I once entered Hanani's office, to find him gazing at a complex looking diagram that he had previously sketched on his office blackboard. "What is it that you are solving," I humbly asked. "Not really solving," Hanani smiled at me while responding: "It is the Four-Color Conjecture again. I left it to rest for forty years. Now I can afford the pleasure to devote what years and strength still left in me to struggle with a problem that I will most likely not live to see solved." "You, however," he proceeded, waving a finger at me in warning, "you, do not dare come anywhere near that problem. It is poison, potentially lethal for young professional careers."

Ironically, when we had that conversation, the Four-Color Conjecture had already been solved, with the solution not yet announced. Hanani was there to see the proof published, a few months later, with his own old Ph.D. result on the reference list.

Poison for young careers? Not necessarily. I would not recommend the CDC conjecture as an obsession for young researchers, but it can well be a very stimulating and rewarding challenge to deal with, alongside other, more modest problems. The quest is not solely about the destination. It is mostly about the journey. Yes, they say there is a priceless precious trophy, hidden at the end of the road. We may reach it, or not. Meanwhile, let us calmly walk the trail and enjoy the magnificent scenery.

C.-Q. Zhang's book will make a very helpful companion and guide along that journey.

Michael Tarsi
Tel-Aviv, Israel

Preface

The Circuit (Cycle) Double Cover Conjecture (CDC conjecture) is easy to state: *For every 2-connected graph, there is a family $\mathcal{F}$ of circuits such that every edge of the graph is covered by precisely two members of $\mathcal{F}$.* As an example, if a 2-connected graph is properly embedded on a surface (without crossing edges) in such a way that all faces are bounded by circuits, then the collection of the boundary circuits will "double cover" the graph.

The CDC conjecture (and its numerous variants) is considered by most graph theorists to be one of the major open problems in the field. One reason for this is its close relationship with topological graph theory, integer flow theory, graph coloring and the structure of snarks.

This long standing open problem has been discussed independently in various publications, such as G. Szekeres (1973 [219]), A. Itai and M. Rodeh (1978 [119]), and P. D. Seymour (1979 [205]). According to Professor W. T. Tutte, *"the conjecture is one that was well established in mathematical conversation long before anyone thought of publishing it."* Some early investigations related to the conjecture can be traced back to publications by Tutte in the later 1940s.

Some material about circuit covers was presented in the book *Integer Flows and Cycle Covers of Graphs* (1997 [259]) by the author as an application of flow theory. There are several reasons why the author decided to write a follow-up book mainly on this subject. Of course, after more than a decade, some new progress and discoveries have been made. Furthermore, in the previous book, circuit cover is a secondary subject and its presentation is based mainly on the techniques and approaches of integer flow theory. However, not all techniques in this subject rely completely on flow theory, and most material can be approached and presented independent of flow theory. Without using flow theory, 3-edge-coloring of cubic graphs becomes one of the major techniques in this book. Since flow theory does provide some powerful tools and beautiful descriptions in this area, it will be covered in later chapters when readers (especially students) have gained sufficient familiarity with the other approaches that they are prepared for further depth. However, most of the book can be read with little or no knowledge of flow theory.

Most of the basic lemmas and theorems of integer flow theory in this book are presented (many without proof) in an appendix. Readers who are interested in further study of the theory of integer flow are referred to the book *Integer Flows and Cycle Covers of Graphs* [259] for more comprehensive coverage.

Since the main subject of this book is circuit double covers, and a smallest counterexample to the CDC conjecture is cubic, most graphs considered in this book are cubic, and the circuit coverage is restricted to 1 or 2 in most cases. Non-cubic graphs will be considered whenever flow theory is applied and in the presentation of a few other techniques.

While topological techniques offer promising approaches in this field, most results and techniques presented in this book are combinatorial. In order to keep the main theme of the book as focused as possible, the author decided to concentrate for the most part on results and techniques in structural graph theory. Topological results are presented only if either they can be obtained by combinatorial methods or they are needed as background for a combinatorial approach.

Most material presented in this book follows the pioneering survey papers by Jaeger (1985 [130]), and Jackson (1993 [121]), and covers the major topics discussed and presented at three historical workshops *(Barbados, February 25–March 4, 1990; IINFORM, Vienna, January 1991; PIMS at UBC, Vancouver, August 22–31, 2007).*

After the publication of the first book by the author, some changes in notation and terminology were strongly suggested by colleagues. Circuits are commonly defined as connected 2-regular subgraphs, while cycles are defined as subgraphs with even degree at every vertex (even subgraphs). These definitions were originally adapted from matroid and flow theory because a vector of the cycle space of a graph is often called a cycle. In order to be consistent with other popular textbooks and avoid possible confusion, in this book a cycle is an even subgraph. Cycle (even subgraph) double cover, circuit double cover are technically the same (in most cases) because every even subgraph has a circuit decomposition.

A set of appendices collects some fundamental graph theoretical results which are useful in the study of circuit covers.

The Petersen graph has been at the core of much major research in structural graph theory. Families of graphs containing no subdivision of the Petersen graph have been proved to have various strong properties. However, for families of graphs with a Petersen subdivision, either some graphic properties do not hold, or determining whether they do is an extremely hard problem. Many circuit covering problems fall in one of these two categories. For the convenience of readers, an appendix (Section B.2), *A Mini Encyclopedia of the Petersen Graph,* is included in this book.

A section of exercises of varying difficulty is included at the end of each chapter. Some interesting results that are not presented in the main part of the chapters are included in the exercises. Appendix D, *Hints for Exercises*, appears at the end of the book.

For the convenience of readers, basic terminology and notation as well as some special terminology is listed in a glossary.

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Readers may send their suggestions via email to

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or by regular mail to me. Corrections and updated information will be available at the World Wide Web site:

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Cun-Quan 'C.-Q.' Zhang
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