

INDEX

- accelerating reference system, 96
- accelerators, 55
- action, 110, 143
 - adiabatic invariance of, 141–57
 - dimensions of, 107
 - as function of energy, 107
 - variable, 104
- adiabatic
 - change, 141, 144, 145
 - conditions, 141–53, 157–61
 - invariance, 142
 - invariance and gas laws, 144
 - invariant, 150
 - theory, 149–53
- amplitude, 47, 143
- analytic methods, 197
- angle-action variables, 103–16
 - for rotations, 114–16
- angle variable, 104
 - as function of q , 110–12
 - for maps, 209
- angular momentum, 53, 74, 115
- anharmonic oscillator, 101
- approximation methods, 122
- area, 13, 84
- area-preservation, 13
- area-preserving
 - flows, 57–9, 75
 - maps, 163, 181–6, 197, 205–13
 - transformations, 13–18, 84–98
- astronomy, 179
- asymptotically stable, 27
- attractor, 26, 38
 - of logistic map, 200–5, 213
- autonomous system, 1, 23, 24
 - Hamiltonian system, 44–57
- bifurcation, 204, 205
- biological systems, 198
- biology, 8, 11
- birds, 199
- birth, 8
- bouncing ball, 109, 142, 153
- boundary point, 7
- breeding season, 199
- canonical transformation, 84–98, 106, 128
 - group property of, 97
 - infinitesimal, 97, 98
 - necessary and sufficient conditions for, 88
 - time-dependent, 94–6
 - time-independent, 84–94
- Cartesian separability, 28
- centre, *see* elliptic fixed point
- change of coordinate system, 86–9
- change of state, 1, 86, 89
- chaotic motion, 197, 202
 - of Hamiltonian systems, 205, 213, 214
- characteristic equation, 13
- charged particles, 141
 - containment of, 206
- chemical reactions, 1, 10
- circular phase space, 6
- classification of fixed points, 32–6
- commutation, 175
- compact manifold, 217
- competition, 8
- complex conjugate eigenvalues, 14
- complex dynamical systems, 202
- complex plane, 14
- compressible fluid, 3
- configuration, 42, 67
 - space, 42
 - circular, 52
- conjugate
 - momentum, 43, 66, 72, 84
 - variables, 43
- conservative Hamiltonian system, 44–57, 141, 214
- contained motion, 27
- containment of charged particles, 206
- contours, 45, 57
 - for bead on wire, 75
 - for cubic potential, 50
 - for general potential, 52
 - for pendulum, 56, 126
- contraction, 19
- convexity, 70, 78
- coordinate, generalized, 43
- cubic potential, 49, 50, 60

- Cremona Map, 208
 cycloid, 79
 cylindrical phase space, 52–7, 105
 cylindrical representation, 52
- damped vertical pendulum, 40
 damping, 173
 death, 8
 degree of freedom, 24, 42
 dependent variable, 90
 difference
 equation, 9
 map, 9
 dilation, 18–21
 discrete time, 197, 198
 double zero, 5
- Earth, 122, 206
 ecology, 8
 economic system, 198
 eigenvalues, 14, 15, 33, 34
 electrical circuit theory, 179
 electronic system, 198
 elliptical phase curve, 148
 elliptic fixed point, 22, 31–3, 36, 47, 51, 55
 elliptic map, 182, 183
 energy, 44, 114, 143
 envelope, 68
 equation of change, *see* equation of motion
 equation of motion, 1, 23
 for bead on wire, 66
 formulation of, 72–8
 equilibrium, 3, 26, 27, 51
 existence theorems, 216–18
 exponential, 8
 decay, 164
 extinction of species, 181
- fast perturbations, 153–7
 first-order
 corrections, 129, 133
 perturbation theory, 122–33
 first-order systems, 1–11
 autonomous, 1–11
 forced, 163–5
 linear, 5, 191–3
 perturbation of, 123–5
 with time-dependent parameters, 165–70
 fixed points, 3, 26, 27, 54
 classification of, 32–6
 determination of, 36, 37
 of Hamiltonian systems, 45
 of logistic map, 199–204
 non-isolated, 54
 oscillatory stable, 169
 oscillatory unstable, 169
- fixed points, (continued)
 overstable, 169
 stable, 3, 169
 unstable, 3, 169
- flow, 34
 area-preserving, 57–9, 76
 linearized, 36
- fluid, 3, 25
 compressible, 3
- forced linear oscillator, 170–3, 193
 forced systems, 163–5, 170–3, 193–6
 forcing frequency, 172
 formulation of equation of motion, 72–8
 freely rotating body, 53, 115, 134
 free particle, 69, 177
 free rotation, 52–4
 frequency, 143
 fundamental particles, 205
- gas laws, as example of adiabatic invariance, 144
- Gaussian numerical integration, 57
- generalized
 coordinate, 43, 67, 84
 hyperbolas, 30
 parabolas, 28
 rotation, 16
 velocity, 67, 72
- generating functions, 87–92, 98
 relation between types, 89–92
 for transformation to angle-action representation, 112–14
- generation, 202
- generator, 98
- gravitational
 field, 77
 potential energy, 73
- Hamiltonian, 43, 68, 70
 in angle-action representation, 103, 106–10
 for bead on wire, 76
 as generators of area-preserving flows, 57–9
 for pendulum, 54, 73
 for pendulum with moving support, 76, 77
 periodic, 52–7, 103–16, 213, 214
 perturbed, 122
 time-dependent, 58, 141–57, 163, 213, 214
 time-dependent transformation of, 96, 97
 time-independent transformation of, 84–93
 unperturbed, 122
- Hamiltonian contours, 42, 43
 for bead on wire, 75

- Hamiltonian contours, (continued)
 for cubic potential, 49, 50
 for general potential, 51
 for pendulum, 54, 55
 Hamiltonian formulation, 67
 Hamiltonian function, *see* Hamiltonian
 Hamiltonian mechanics, 10
 Hamiltonian systems, 8, 27, 42–65, 205
 chaotic motion of 213, 214
 conservative, 44–57
 equilibrium of, 51
 fixed points of, 51
 regular motion of, 213, 214
 Hamiltonian's equations, 43, 57
 harmonic oscillator, *see* linear oscillator
 Hénon, M, 208
 hyperbolic
 fixed point, 30, 33, 35, 48, 51, 55, 211
 map, 182
 ice skater, 158
 identity transformation, 88, 91
 improper node, 33, 34, 36
 impulse, 177, 186, 190
 convention, 178
 definition of, 178
 propagator for, 177–9
 incompressible flow, 58
 infinitesimal
 canonical transformation, 97, 98
 rotation, 102
 translation, 98
 insects, 199
 integrable systems, 214
 invariant
 adiabatic, 150
 curves, 208–13
 regions, 105
 sets, 3, 45
 inverse propagator, 166, 175
 irrational numbers, 185
 irregular motion, 202
 islands, surrounding fixed points, 210–13
 Jacobian, 85
 Jupiter, 122, 206
 Kinetic energy, 43, 67, 72, 77
 Lagrangian equation of motion, 67, 70–2
 Lagrangian formulation, 67
 Lagrangians, 67–78
 for bead on wire, 74
 for free particle, 69
 for pendulum, 73
 for pendulum with moving support,
 76–7
 lasers, 141
 Legendre transformation, 67–70, 78
 librate, 47
 libration, 55–7, 103
 period of, 57
 limit cycle, 37, 38
 linear
 area-preserving maps, 181–91, 197
 area-preserving transformation, 13–18,
 177
 differential equation, 5
 equation of motion, 163
 Hamiltonian systems, 45–9
 homogeneous system, 167
 map, 12, 179–91
 motion, 32
 linearized flow, 36
 linear oscillator, 46, 47, 64, 101, 102, 121,
 127, 134, 194
 angle-action variable for, 113
 forced, 170–3
 with periodic impulses, 189, 190
 propagator for, 175, 176
 with slowly varying frequency, 144–9
 linear
 repulsive force, 48, 49
 restoring force, 190, *see also* quadratic
 potential
 linear systems, 5, 28
 conservative, 45–59
 first-order, 163–70, 191–3
 linear transformation, 12–21
 area-preserving, 13–18, 177
 Type 1, 14
 Type 2, 14
 Type 3, 15
 Liouville's Theorem, 57–9
 local analytic solutions, 26
 logistic equation, 8, 9, 200
 logistic map (ping), 199–205, 214
 attractors of, 200–5, 213
 many degrees of freedom, 55
 map, 197
 and discrete time, 198, 199
 linear area-preserving, 163, 181–6,
 197, 205–13
 logistic, 199–204
 n th order, 198
 non-linear, 197–215
 and periodic conditions, 198
 second generation, 203
 Type 1⁺, 182
 Type 1⁻, 182
 matrix representation, 12
 maxima (of potential), 51
 mean motion Hamiltonian, 156
 mechanical systems, 198
 microphone capacitor, 189

- minima (of potential), 51
- molecular
 - motion, 55
 - rotation, 55
- momentum, 72, *see also* conjugate momentum
- Morse potential, 135
- motion
 - contained, 27
 - in rapidly oscillating field, 153–7
- multiplication rule for propagators, 166, 175
- natural boundaries, 7, 8, 30, 107, 114
- natural frequencies, 172
- neuroelectrical activity, 65
- Newtonian
 - formulation, 71
 - mechanics, 45
 - systems, 1, 23, 42, 43, 188, 205
- Newton's equation of motion, 24, 45
- perturbation theory applied to, 139
- node
 - improper, 33, 34, 36
 - stable, 28, 33, 34
 - unstable, 29, 33, 34
- non-inertial reference frame, 58, 59
- non-linear, 121, 173, 179, 197, 213
- open interval, 3
- order of system, 23
- oscillate, 47
- oscillation, *see* libration
- oscillatory motion, 51, 189
- overstable, 169, 183
- parametric
 - changes, 190
 - resonance, 163, 186–91
- parasite, 39
- particle accelerator, 141
- particle, falling freely, 25
- pendulum, 26, 42, 54, 55, 117, 136, 157
 - contours for, 56, 126
 - damped, 40
 - Hamiltonian for, 55, 73, 74
 - Lagrangian for, 73, 74
 - with moving support, 76
 - perturbation theory applied to, 126, 130–3
 - with rapidly moving support, 156
- period, 55–7, 61
 - for libration, 57
 - propagator, 168, 186
 - for rotation, 57
- periodic
 - conditions, 163, 179–91
 - conditions giving rise to maps, 198
 - forces, 186–91
 - periodic (continued)
 - motion, 105
 - potential, 115
 - system, 169
 - time-dependent parameters, *see* periodic conditions
 - periodicity, 151
 - perturbation, 122
 - fast, 153–7
 - slow, 144–53
 - small, 5, 124
 - perturbation theory, 122–33
 - for adiabatically varying systems, 146
 - for conservative Hamiltonian systems, 125–33
 - for first-order systems, 123, 124
 - perturbed
 - Hamiltonian, 122
 - motion, 127
 - phase curves, 133
 - phase curve, 23, 24
 - for pendulum, 55
 - perturbed and unperturbed, 133
 - phase diagram, 25, 119
 - for conservative Hamiltonian system, 45
 - simplified, 45
 - phase flow, 1, 23, 25
 - as an infinitesimal canonical transformation, 98
 - of second-order system, 25, 26
 - phase portrait, *see* phase diagram
 - phase space, 1, 23, 24, 42, 43, 198
 - circular, 6
 - cylindrical, 52–7, 105
 - transformation in, 84–98
 - phase velocity, 1, 23, 24, 43, 57
 - of conservative Hamiltonian system, 44
 - physical
 - acceleration, 24
 - momentum, 43
 - velocity, 24, 67
 - planetary motion, 55, 122
 - plasma, 55
 - pocket calculator, 214
 - polar separability, 30
 - population changes, 1, 8, 9, 199
 - potential, 3, 43
 - barrier, 51
 - energy, 43, 72, 77
 - periodic, 115
 - maxima of, 51
 - minima of, 51
 - propagator, 163, 165–70, 173–91
 - of first order system, 165–76
 - for free particle, 177–9
 - for impulsive motion, 177–9
 - inverse, 166, 175
 - for linear oscillator, 175, 176

- propagator, (continued)
 for linear repulsive force, 176, 177
 multiplication rule, 166, 175
 period, 163, 179, 186
 for second-order systems, 173–91
- quadratic
 area-preserving map, 205–13
 barrier, 48
 impulse, 207
 potential, 46, 190, *see also* linear oscillator
 oscillatory behaviour, 25
- rapidly oscillating
 conditions, 141, 153–7, 161, 162
 force, 142
 Hamiltonians, 161–3
- rational numbers, 185, 211
- real and distinct eigenvalues, 14
- real and equal eigenvalues, 15
- reflexion, 16, 20
- regular motion, 208
 of Hamiltonian systems, 213, 214
- relativistic particle, 83
- representation
 matrix, 12
 (q, p) , 67, 76
 (q, \dot{q}) , 67, 76
 transformation between different, 84–98
- resonance, 173
 condition, 172
 parametric, 186–91
- rotating reference frame, 58
- rotation, 6–8, 16, 20, 52–7, 63, 103, 114–16
 free, 52–4
 period of, 57
 uniform, 6
- saddle point (of potential), 51
- second generation map, 203
- second-order
 differential equation, 24
 system, 12, 23–41, 163, 170–91, 193–6
- separation of variables, 28–32
- separatrix, 48, 48–52, 55, 61, 103, 126, 129, 133, 211
- shear, 17
- Siegel C.L., 208
- simple harmonic oscillator, *see* linear oscillator
- simplest variables, 103–6
- simplified phase diagram, 45
- slow perturbation, 144–53
- small perturbation, 5, 124
- smooth periodic function, 191
- social systems, 198
- solar system, 42, 122, 206
- spiral point, 30–6
 squeeze, 16, 20
- stability, 26–8, 163, 190
 asymptotic, 27
 bands, 190
 of fixed points, 3, 26, 27
 strong, 27, 29, 169
 structural, 5, 6, 38
- stable
 elliptic point, 185
 fixed point, 3, 27, 202
 node, 28
 population, 9
 star, 29
- standard
 form, 15
 shear, 17
 squeeze, 16
 transformation, 15, 19
- star
 stable, 29, 33, 35
 unstable, 29, 33, 35
- state, 1, 23
- storage ring, 205
- strange attractors, 213
- stroboscope, 169
- strongly stable, 29, 51
 fixed point, 27, 29, 31, 169
 invariant set, 38
 spiral point, 31
- structural instability, 5, 185
- structurally unstable, 185
- structural stability, 6, 38
- Sun, 122
- swing, 189
- system
 conservative, 44–57
 first-order, 1–11
 integrable, 214
 of order n , 23, 24, 198
 second-order, 23–41, 43
- Taylor expansion, 36, 122
- terminating motion, 6, 61, 124
- time-dependent
 conditions, 1, 57, 163, 165
 forces, 58
 Hamiltonians, 58, 59, 152
 parameters, 163, 165
 transformations, 94–7
- time-independent
 Hamiltonians, 44–65, 103, 125–33, 152
 transformations, 84–94
- trace, 13

- transformation
 - to angle-action representation, 103–16
 - area-preserving, 13–18, 84–98
 - canonical, 84–98
 - as change of coordinate, 86, 87
 - as change of state, 86, 89
 - with dilation, 18–21
 - identity, 88, 91
 - invertible, 13
 - Legendre, 67–70
 - linear, 12–21
 - linear area-preserving, 13–18, 177
 - between (q, \dot{q}) and (q, p)
 - representation, 67–70
 - standard, 5
 - time-dependent, 84–94
 - time-independent, 94–7
 - Type 1, 14, 33
 - Type 2, 14, 33
 - Type 3, 15, 33
- turning point, 51
- unconstrained Newtonian system, 24
- uniform
 - force, 46
 - rotation, 6
- unit square, 15
- unperturbed
 - Hamiltonian, 122
 - motion, 127
 - phase curves, 133
- unstable
 - fixed point, 3, 27, 169
 - hyperbolic point, 183
 - node, 29
 - spiral point, 31
- velocity field, 3
- velocity function, 1, 2, 23, 36
 - which is explicit function of time, 163
- Venus, 122
- vertical
 - acceleration, 77
 - pendulum, 54, 55, *see also* pendulum
- vibration, *see* libration
- winding number, 185, 211