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Ian Percival and Derek Richards
Frontmatter
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Introduction to Dynamics

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PREFACE

Modern dynamics owes as much to Poincaré and Liapounov as to Lagrange and Hamilton, so we introduce Hamiltonian dynamics through the qualitative theory of differential equations and we highlight the geometry of phase curves and the theory of stability. Each subject, from the elementary theory of first-order systems, up to the discoveries on chaotic motion in recent decades, is introduced through simple examples. The mathematical background required of the reader is confined to 2×2 matrices, ordinary differential equations and the calculus of two variables (apart from appendix 1). Some knowledge of elementary Newtonian mechanics would be helpful, but we include other applications, including the dynamics of biological populations.

For simplicity we restrict our attention to first- and second-order systems and to Hamiltonian systems with one degree of freedom. This approach is not nearly so restrictive as one might think and enables us to introduce to undergraduates many important ideas that have previously been confined to graduate teaching or research.

The stronger connexions between the chapters are illustrated in the diagram at the end of the preface, but more advanced students or research workers should find that they can usefully dip into the book with a little cross-reference.

Very many colleagues and students have helped us. In particular we thank Brian Chirgwin, Barry Hughes, Alan Jeffrey, Mike Simpson and the referees of the Cambridge University Press for their advice and criticism of early drafts. We are grateful to Michel Hénon for lending us the originals of figures for chapter 11, to the editor of the *Quarterly of Applied Mathematics* for permission to publish them and to John Greene and Mike Lieberman for helping to clarify some of our ideas on that chapter. Like so many people, we are grateful to Joe Ford for infecting us with his enthusiasm for modern dynamics. We could not have finished the book so quickly without diligent typing assistance from Frances Thomas and assistance with the computing from Alf Vella. We could not have written it at all without the encouragement of Jill and Helen, to whom this book is dedicated.

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Stronger connexions between chapters

