# *Partial differential equations*

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CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Victoria 3166, Australia

Originally published in German as *Partielle Differentialgleichungen*
by B.G. Teubner, Stuttgart, 1982 and © B.G. Teubner, Stuttgart 1982

First published in English by Cambridge University Press 1987
as *Partial differential equations*


Reprinted 1992

*British Library cataloguing in publication data*

Wloka, J.
Partial differential equations

1. Differential equations, Partial
I. Title II. Partielle Differential-
gleichungen. *English*
515.3'53 QA374

*Library of Congress cataloguing in publication data*

Wloka, Joseph.
Partial differential equations.
Translation of: Partielle Differentialgleichungen.
Includes index.
1. Differential equations, Partial. I. Title.
QA374.W5613 1987 515.3'53 86-18817

ISBN 0 521 25914 2 hardback
ISBN 0 521 27759 0 paperback

Transferred to digital printing 2002

TM

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For Brigitte

Cambridge University Press
978-0-521-27759-4 - Partial Differential Equation
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# *Preface*

Boundary value problems are the subject of this book. All boundary value conditions for elliptic differential operators are given, using the Lopatinskiĭ–Šapiro condition (= covering condition), which lead to the normal solvability of a boundary value problem. The variational method is also presented in detail, and questions of its connections with general elliptic theory considered. Those parabolic and hyperbolic equations for which the right-hand side (derivatives with respect to $x$) is an elliptic differential operator are considered, and the knowledge about elliptic operators is used in order to obtain insight into the solvability and regularity properties of the solution for mixed problems.

I have chosen a form of the Lopatinskiĭ–Šapiro condition which allows us to test immediately, whether or not given boundary value conditions satisfy it. It appears that all classical boundary value problems satisfy it, the examples are worked through individually.

In order not to overexpand the compass of the book, and to maintain its introductory character, I have not considered pseudo-differential operators; all the same I have proved the main theorem for elliptic boundary value problems by means of pseudo-differential operators – without calling them such.

Before the discussion of differential equations there is an introductory chapter on distributions and Sobolev spaces; here I have proceeded in an elementary way, working with the Fourier transformation, and not using interpolation theorems. This is possible without further assumptions as long as one remains inside $L^2$-theory. I have not considered the $L^p$-theory; this comes into its own for non-linear equations, see for example Lions [3], while in the linear case it does not bring any essentially new insights.

I have looked very precisely into the differentiability properties of the frontier $\partial\Omega$; in the end we want to solve the Dirichlet problem on the square as well as on the circle, and the proofs are not essentially simpler for the $C^{\alpha}$-theory.

Among the many procedures for the practical solution of a partial differential equation I have singled out the difference process – its appeal lies in its simplicity: derivatives are replaced by difference quotients and so we obtain a system of linear equations, which in the main we can solve by the usual methods. In the last chapter I wish less to lead into modern methods, than to transmit the feeling to the reader that it is actually possible numerically to solve partial differential equations.

The reader ought to be familiar with the language of functional analysis, at about the level of the books of Heuser [1], or Wloka [1]. He may find the basic theorems of functional analysis relevant for analysis, such as the Hahn–Banach, Banach–Steinhaus and Riesz theorems, the open mapping theorem..., collected and proved on pp. 12–27 in L.H. Loomis, *An introduction to abstract harmonic analysis*, New York (1953). In separate sections I have thoroughly considered less familiar material such as, for example, the theory of Fredholm operators, Gelfand triples, abstract Green solution operators, the Schauder fixed point theorem and the Bochner integral. In this way I hope to spare the reader a time-consuming hunt through the literature. Another aim, which I have included in these functional analytic sections, is wherever possible to replace hard analysis by soft analysis, and so restricted the difficult estimation machinery to an unavoidable minimum. In this way I also believe that I have given the reader a better view of the connections and possible generalisations.

I owe a very heavy debt of gratitude to R. Mennicken, G. Bauer and B. Sagraloff from Regensburg, to my students J. Benner, R. Janssen, R. Rath, and to Miss Inga Haecks. They have read the complete manuscript critically and carefully, and have provided many important remarks and improvements. I am particularly grateful to M. König for assistance in the laborious proof reading of the German edition.

I thank Professor Dr G. Köthe for encouraging me to write this book, and the publishers both for their assistance in the preparation of the manuscript and cooperation in its presentation.

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*Note added in the English translation*

I have made some corrections and slight changes, especially in §§ 14 and 15. To English-speaking readers, instead of the books of Heuser [1] and Wloka [1] (see above) I would recommend those of Schechter [1] or Taylor [1] as an introduction to the language of functional analysis.

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