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PREFACE

For there is no man can Write fo warily,
but that he may fometime give Opportunity
of Cavilling, to thofe who feek it.

John Wallis, A treatife of Algebra, 1685.

From rather modest beginnings the British Combinatorial Conference has grown into an established biennial international gathering. A successful format for the Conference has been established whereby several distinguished mathematicians are each invited to give a survey lecture at the Conference and to write a paper for the Conference Volume, which is published in time for the start of the meeting. The present volume contains eight of the nine invited papers for the Ninth Conference held at the University of Southampton, 11-15 July 1983.

Between them the papers cover a broad range of combinatorics. The all-pervading subject of graph theory appears in a number of the papers. It is the central feature of the one by J.C. BERMOND and his co-authors in which they survey those results concerning diameter and connectivity in graphs and hypergraphs of importance for interconnection networks. Graph theory is also used by J.M. HAMMERSLEY in his study of the Friendship Theorem and the Love Problem. His paper looks back to classical mythology with references to Narcissus, but in producing it he has made use of the latest technology in the form of the Oxford University Laser-comp typesetting facility. Perhaps the day is not far off when it will become routine for authors to produce their papers by such means. Other papers using graph theory are those of Schrijver and Shult mentioned below.

J.W.P. HIRSCHFELD writes on Maximal Sets in Finite Projective Spaces, a topic with applications in statistics and coding theory. He has gathered together a huge number of equations and inequalities from a wide literature.

For half a century research on permanents was overshadowed by the van der Waerden conjecture. Recently two proofs of it were obtained independently by Falikman and Egorychev for which work they were jointly awarded one of the Fulkerson Prizes in Discrete Mathematics in August 1982. The paper by A. SCHRIJVER surveying this and related work is, therefore, especially timely. A very different problem which has also been solved recently is that of characterizing the Lie incidence geometries,

which are associated with finite simple groups. The solution involves the patching together of results of many researchers and E.E. SHULT'S paper should help to make the subject more accessible.

J. Howard Redfield was a polymath who earned his living as an engineer. From an early age he showed exceptional linguistic ability, later becoming familiar with most European languages as well as some Asian and African tongues. When he turned his attention to combinatorics he produced a paper (published 1927) the value of which was not generally to be recognised by professional combinatorialists until some thirty years later. The recent discovery of a second (unpublished) paper by Redfield shows that he had continued his pioneering work and had obtained further results on enumeration much earlier than other writers. J. SHEEHAN looks at the material in the two Redfield papers and its relation to the work of others including Pólya, Read and himself. Redfield's first paper had opened with the very perceptive statement: "In view of the similarity which will be admitted to hold between the subject matters of the Theory of Finite Groups and of Combinatory Analysis, it is somewhat surprising to find that in their literatures the two branches have proceeded on their separate ways without developing their interrelationship,...". Half a century later this similarity is now well recognized and also extends to certain infinite groups some of which are studied by R.P. STANLEY in his paper on $GL(n, \mathbb{C})$ for Combinatorialists. Nowadays some group theory is part of the stock-in-trade of almost every mathematician, but the same cannot be said of quasigroup theory. The paper by C.C. LINDNER, written in a very readable style, is recommended to anyone wishing to learn something about the subject.

Reference to the indices will show that although the papers are all on different topics some ideas and some names appear in more than one of them. In some cases the definition of a term used but left undefined by one author may be found in one of the other papers.

In editing this volume I have received assistance from the referees of the papers and from many other colleagues, to all of whom I express my grateful thanks. In particular my eagle-eyed colleague Gareth Jones read some of the papers and helped me to minimise the number of misprints and other errors in the volume now before you. The secretarial staff of the Faculty of Mathematical Studies, University of Southampton, and especially Margaret Youngs with her speedy and accurate

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Preface

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typing, also provided valuable help. The London Mathematical Society agreed to the inclusion of this volume in their Lecture Note Series and also provided some financial assistance for the Conference. A book such as this has to be produced to a tight schedule and so I am very happy to thank the eight authors for their cooperation in producing their papers within the necessary time limits, and to thank Simon Capelin who has handled the editorial arrangements at Cambridge University Press.

Finally my thanks to Norman Pearce for his smooth driving of the coach on which most of this preface was composed during a weekend visit to study the industrial archaeology of Kent and East Sussex.

A special issue of 'Ars Combinatoria' will contain contributed papers of the Conference.

Hastings,

26 March 1983

E.K.L.

The paper by V.T. SÓS, Irregularities of partitions, was received at a very late stage and it has not been possible for it to be edited as thoroughly as the other papers nor for it to be indexed. The paper discusses the relationship between some classical results on uniformly distributed sequences and Ramsey-type theory. Some addenda and errata to the paper have been added on page 246.

Southampton,

13 April 1983

E.K.L.