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A. R. PATERSON

Lecturer in Mathematics, University of Bristol



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Preface

An author makes his excuses in a preface, so here are mine. For a number of years at Bristol University we tried to find a suitable text to introduce fluid dynamics to second year mathematics students, and failed. The modern texts with the ‘right’ attitude to the subject were too hard for a first course, the older texts were dominated by potential theory and unrealistic examples. This text has been tried in draft form for several years on our students, and has been judged ‘hard, but interesting’. New work in mathematics is always hard, but I believe that the level chosen here is a suitable one.

I apologise to my colleagues for the gross over-simplification of their work and their subject which is committed in this book. And also for the errors and misapprehensions – students, beware! all texts have mistakes in them. I thank my colleagues for helpful comments and discussions over many years; I also thank a succession of seminar speakers for maintaining my awareness of the full range of fluid dynamics.