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Solitons

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To JUDITH

CONTENTS

PREFACE

The theory of solitons is attractive; it is wide and deep, and it is intrinsically beautiful. It is related to even more areas of mathematics and has even more applications to the physical sciences than the many which are indicated in this book. It has an interesting history and a promising future. Indeed, the work of Kruskal and his associates which gave us the inverse scattering transform is a major achievement of twentieth-century mathematics. Their work was stimulated by a physical problem and is also a classic example of how computational results may lead to the development of new mathematics, just as observational and experimental results have done since the time of Archimedes.

This book originated from lectures given to classes of mathematics honours students at the University of Bristol in their final year. The aim was to make the essence of the method of inverse scattering understandable as easily as possible, rather than to expound the analysis rigorously or to describe the applications in detail. The present version of my lecture notes has a similar aim. It is intended for senior students and for graduate students, phyicists, chemists and engineers as well as mathematicians. The book will also help specialists in these and other subjects who wish to become acquainted with the theory of solitons, but does not go as far as the rapidly advancing frontier of research. The fundamentals are introduced from the point of view of a course of advanced calculus or the mathematical methods of physics. Some knowledge of the elements of the theories of linear waves, partial differential equations, Fourier integrals, the calculus of variations, Sturm-Liouville theory and the hypergeometric function, but little more, is assumed. Also some familiarity with the elements of the theories of water waves, continuous groups, elliptic functions, one-dimensional wave mechanics and Hilbert spaces will be useful, but is not essential. References are given to help those readers who have not learnt these topics. The diverse applications

of the theory of solitons are only mentioned briefly in the main text and in the problems.

Simplicity and concrete applications are emphasized in order to make the material easily assimilable. The more difficult sections, paragraphs and problems are preceded by asterisks; it is suggested that they are omitted on a first reading of the book. Further reading is recommended to cover results which are only quoted here and to offer more detailed treatments. The equations are numbered in each section separately. An equation within the same section is referred to simply by its number; in a different section by its section and equation numbers; and in a different chapter by its chapter, section and equation numbers. The sections and figures are numbered in each chapter separately and are referred to similarly.

I am grateful to Miss Sarah Trickett for computing the solutions of the Korteweg-de Vries equation used to draft Figures 4.6, 4.7 and 4.8; to Drs A. Davey, D.H. Griffel, R.S. Johnson, I.M. Moroz and A.R. Paterson for criticisms of an earlier draft of this book; to Mrs. Tina Harrison for drawing the figures; and to Mrs Nancy Thorp for typing the text — its quality is immediately evident to the reader although my last-minute changes may not be.

P.G. Drazin
Bristol
February 1983