1

Graph theory and anthropology

This new mathematics (which incidentally simply gives backing to, and expands on, earlier speculative thought) teaches us that the domain of necessity is not necessarily the same as that of quantity.

Claude Lévi-Strauss, "The mathematics of man"

Anthropology is fundamentally the study of sets of social and cultural relations whose diversity and pervasiveness is illustrated by such terms as "exchange," "hierarchy," "classification," "order," "opposition," "mediation," "inversion," and "transformation." The analysis of these relations always presupposes models of some kind, implicit if not explicit, informal if not formal. The models are usually defined in ordinary language, but with results that are not always satisfactory in matters of descriptive adequacy, insight, and communicability. The question thus arises as to whether, in many contexts, mathematical formulations might not be helpful; and if so, what kind of mathematics.

Some time ago, in his essay "The mathematics of man," Lévi-Strauss emphasized the suitability of the various forms of modern mathematics as a source of structural models in anthropology:

In the past, the great difficulty has arisen from the qualitative nature of our studies. If they were to be treated quantitatively, it was either necessary to do a certain amount of juggling with them or to simplify to an excessive degree. Today, however, there are many branches of mathematics – set theory, group theory, topology, etc. – which are concerned with establishing exact relationships between classes of individuals distinguished from one another by discontinuous values, and this very discontinuity is one of the essential characteristics of qualitative sets in relation to one another and was the feature in which their alleged "incommensurability," "inexpressibility," etc., consisted. (Lévi-Strauss 1955:586)

In contrasting the old and the potentially new forms of mathematical thinking in anthropology, Lévi-Strauss observed by way of example that we should be "less concerned with the theoretical consequences of a 10 per cent increase in the population in a country having 50 million inhabitants than with the changes in structure occurring when a 'two-person household' becomes a 'three-person household'" (Lévi-Strauss 1955:586). One immediately thinks of a triangle whose points represent the members of the household and whose lines represent their relations to one another. This characterization of structural anthropology serves not only to distinguish mathematical models from statistical ones, but also evokes graph theory in particular. Since the example refers to social structure and suggests a pictorial representation, it is intuitively meaningful to most anthropologists, for it is comparable to familiar anthropological graphs such as genealogical trees or drawings of trade networks. By the term "structural models" we shall mean not only just such graphs, but go beyond the mere representation of social relations to consider as graphs a variety of empirical phenomena whose underlying structural properties can be elucidated through the application of a well-developed body of relational concepts and theorems.

Our aim is to introduce graph theory as a comprehensive structural model, or family of models, in cultural and social anthropology. Graph theory is a branch of finite mathematics that is both topological and combinatorial in nature. Because it is essentially the study of relations, graph theory is eminently suited to the description and analysis of a wide range of structures that constitute a significant part of the subject matter of anthropology, as well as of the social sciences generally. We have in mind not only social networks, whose underlying graph theoretic basis is easily recognized, but a variety of social, symbolic, and cognitive structures as well. Belief and classification systems turn out to be no less graphical than communication and exchange networks. By showing this to be so, we hope not only to provide a standard language for the use of graphical models, but also to enlarge the field of structural analysis in anthropology.

In addition to the intrinsic advantages of graph theory as a structural model, there are significant interactions between graphs and relations, matrices, duality laws, and groups that increase enormously the diversity of potential empirical applications. Thus, for example, studies of the logical properties of social structure, matrices of exchange relations, transformations in myths, and permutations in symbolic systems are commonly but often unknowingly based on graph theory. When this theoretical basis is made fully explicit, analysis is usually clarified and often enhanced. Fortunately, graph theory is also relatively self-contained. Thus this book presupposes no background in it and is readily accessible to the nonmathematical reader.

For convenience of exposition, the organization of the book is dictated more by graph theoretic than by anthropological considerations. The presentation is cumulative: Chapter 2 starts with the simplest type of graph which will be modified, extended, and reinterpreted in successive chapters to yield an array of structural models. The examples brought to bear are in each case anthropologically real, as well as diverse and suggestive of further research. They are designed to convey not only a general understanding of the models, but also a concrete idea of how to apply them. This combina-

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tion of theoretical exposition and meaningful exemplification will provide a basic introduction to a large and at present explosive field of applicable mathematics.[1] We will begin that introduction with a brief informal presentation of some of the key concepts of graph theory.

Graphs informally

A graph is a structure consisting of points joined by lines. Fig. 1.1 shows an ordinary graph *G* which depicts a subsystem of a larger exchange system in a Papuan village (Schwimmer 1973). The points represent households, and the lines represent regular and symmetrical gifts of cooked taro given by women on behalf of each household.

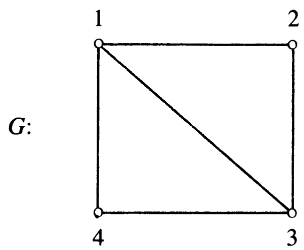

Fig. 1.1. A graph *G* of a New Guinea exchange system.

These dyadic gift relations are a sign of social intimacy, and their concatenation serves to connect indirectly all 22 households in the village, not just these 4. The indirect connections are especially significant because they are used to make requests for economic and political assistance. Thus, in Fig. 1.1 the indirect connection is a 2-step path when household 2 asks 3 (or 1) to ask 4 for a favor of some sort.

By representing a system of this type as a graph, one can study certain formal properties of social structure, together with their empirical implications. In this particular system, for example, the relative power or influence of individuals (households) depends upon the structure of mediated communication, a property that is precisely defined by the graph theoretic concept of betweenness. This refers to the frequency with which one point occurs on the paths joining all other pairs of points. Betweenness is one of the numerous definitions of centrality in a graph, and centrality, in turn, is one of a still larger set of concepts applicable to notions of communication and subordination in social systems.

We note that it makes no difference how this graph is drawn, since our only concern is with the pattern of relations, with how the points are joined by the lines. The graph of Fig. 1.2 is thus isomorphic to that of Fig. 1.1.

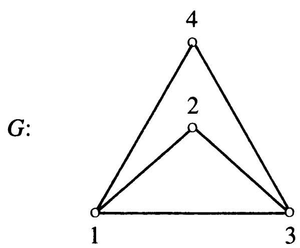

Fig. 1.2. An equivalent representation of the graph in Fig. 1.1.

[1] For more graph theory, see the text by the same title (Harary 1969). For a review of anthropological applications see Hage (1979a).

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Social relations are not always all or none, present or absent, but may be positive or negative. By assigning + and − signs to the lines of a graph *G*, represented for convenience by solid and broken lines, respectively, we get a signed graph *S*. The one in Fig. 1.3 depicts a Melanesian system of competitive exchange based on Michael Young's (1971) monograph, *Fighting with food*. The points of S represent patrilineal clans or clan segments; the positive lines represent the relation of *fofofo,* or food friend, and the negative lines represent the relation of *nibai,* or food enemy.

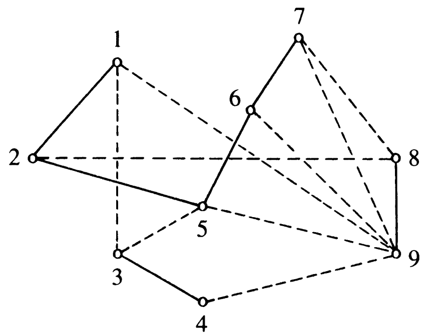

Fig. 1.3. A signed graph *S* of a Massim competitive exchange system.

In this system *nibai* make competitive gifts of forbidden food, *niune*, to each other. This food cannot be consumed; it must be passed on to a *fofofo*. Thus clan 1 gives food to clan 3, which passes it on to clan 4; clan 9 gives to 4, which passes it on to 3. The overall pattern of food friend/enemy relations determines alignments in disputes.

There is an expectation based on theories of structural balance and clustering that relations in such systems will combine in consistent ways, or equivalently, that the group will be divisible into cliques or opposing coalitions. By modeling alliance and sentiment structures as signed graphs, we can discern consistency and contradiction in sets of relations, and in some cases we can determine, by means of simulation, how such patterns are generated.

Not all social relations are symmetric. A directed graph *D* is one in which the lines have arrows, thus permitting the representation of both asymmetric and nonsymmetric as well as symmetric relations. A theoretically interesting illustration, which shows what can happen when a projected six-person household becomes an eight-person household, comes from French literature.

Michael Oppitz (1975) in his book on structural anthropology (felicitously entitled *Notwendige Beziehungen*) introduces Lévi-Strauss's (1969) theory of marriage exchange by showing that it was anticipated over 150 years earlier by the Marquis de Sade. In *Les 120 journées de sodome*, four libertines plan to celebrate an elaborate and ongoing series of orgies at a castle owned by one of them. In order to solidify their relations, one member of the group, the Duke of Blangis, proposes a triple marriage alliance in which he will give his daughter Julie to his friend the Président de Curval, the Président will give his daughter Adelaide to a third individual, a M. Durcet, and M. Durcet will give his daughter Constance to the Duke. This alliance having been agreed upon, the fourth member of the group, the Bishop of ——— is accommodated by becoming the spouse of all three women by giving his niece Aline (actually his daughter by his brother's, the Duke's, wife) to all three men. Thus, as Oppitz observes, Sade becomes the "first anthropological theoretician who expressly represents marriage as a system of communication."

The triple alliance among the Duke, the Président, and M. Durcet is an asymmetric form of marriage exchange that corresponds to Lévi-Strauss's generalized exchange: In the simplest case A gives a woman to B, B gives one to C, and C gives one to A. The alliance among these three individuals and the Bishop is a symmetric form of marriage exchange and corresponds to Lévi-Strauss's restricted exchange: In the simplest case A gives a woman to B, and B gives one to A.[2] The directed graph of this system, which incorporates both types of relations, symmetric and asymmetric, is shown in Fig. 1.4.

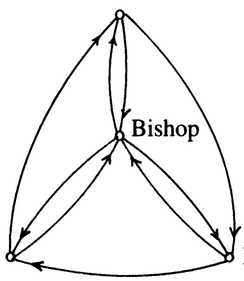

Fig. 1.4. A directed graph *D* of marriage exchange in Sade's *Les 120 journées de sodome*.

One of the most significant advantages of graph theory is that it provides diagrammatic models for the representation of diverse types of structures. For example, the logical analysis of social structure recommended some time ago by Lévi-Strauss (1963a) and subsequently attempted in areas as diverse as Basque studies (Ott 1981) and Melanesian anthropology (Kelly 1974) would be enormously facilitated by using graphs such as the one in Fig. 1.4 to model concepts like symmetry, transitivity, reflexivity.

[2]The terms "generalized exchange" and "restricted exchange" are actually complex and based on multiple criteria, as Barnes (1971) shows in his excellent discussion. For present purposes: "Generalized exchange establishes a system of operations conducted 'on credit.' A surrenders a daughter or sister to B, who surrenders one to C who, in turn will surrender one to A. This is its simplest formula" (Lévi-Strauss 1969:265). "The term 'restricted exchange' includes any system which effectively or functionally divides the group into a certain number of pairs of exchange units, so that for any one pair X–Y there is a reciprocal exchange relationship" (Lévi-Strauss 1969:146).

Actually, a graph need not be represented pictorially, although it is often convenient and intuitively helpful to do so. Instead, a matrix can be used in which each point has a row and a column and in which the entries in the cells are either 1 or 0 to show the presence or absence of a line joining a pair of points. Thus the adjacency matrix A of the graph G in Fig. 1.1 is as follows:

$$A(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

A matrix conveys exactly the same information as a drawing, but has the advantage that it can be algebraically manipulated to reveal higher-order structural properties – for example, the distance between each pair of points. Thus properties such as centrality, clustering, and transitivity in graphs, signed graphs, and directed graphs can be studied just by using matrix methods. Indeed they must be studied in this way when structures become large.

If we use the term graph in its generic sense to include ordinary graphs G, signed graphs S, and directed graphs D, perhaps its most immediate interpretation is as an exchange system like one of those noted so far. However, any of these types of graphs could depict cognitive schemata where the points represent categories and the lines represent relations, say, of contrast, implication, sequencing, and so on. Or they could depict symbolic systems where the points represent sets of cultural beliefs or practices and the lines relations of transformation or symmetry.

One way to think of a transformation is to use the theory of structural duality in graphs. For every type of graph, there is a dual operation that changes it to give another graph with the same set of points and that, when applied twice, results in the original graph. This is structural duality. In the case of a directed graph D, for example, the operation, called taking the converse, consists of reversing the direction of all the arrows to get D'. A very simple example from mythology may be given. In *Naked man,* Lévi-Strauss (1981) describes inverse relations between myths concerning the loon as found on the Pacific and Atlantic coasts. One of these inversions concerns the hierarchical relationship between the loon and another bird, the diver, which Lévi-Strauss represents symbolically as in Fig. 1.5(a), and which we depict as a directed graph D and its converse D' in Fig. 1.5(b).

Besides the operation of taking the converse, there are comparable dual operations for signed graphs and for ordinary graphs called negation and complementation, respectively. The theory of structural duality provides a source of transformation rules for the analysis of symbolic systems and at the same time yields a way of disentangling various meanings of the word "opposite," which carries such a heavy semantic load in structural analysis.

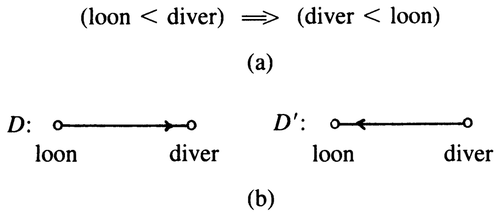

Fig. 1.5. The converse D' of a directed graph D in mythology.

For certain types of problems in anthropology, one is concerned not only with the presence or absence of a relationship, but also with its strength or multiplexity, as is emphasized, for example, in social network studies (Mitchell 1969). Then, when empirically justified, numbers can be assigned to the lines of a graph to give a network N. In the system depicted in Fig. 1.1, for example, the ethnographer determined not only the existence of food exchanges between households, but tabulated the amount of the exchanges as well by making daily observations over a period of several months. This enabled him to discriminate first, second, and third preferential partners on the basis of the relative amounts household A gave to households B, C, and D. Fig. 1.6 shows the resulting network of the graph in Fig. 1.1; the numbering on the lines indicates the degree of preference of one exchange partner for another. By assigning numbers to the lines of a graph or directed graph, it becomes possible not only to take account of the strength of a social relation, but also to study certain aspects of group dynamics using models of information flow in capacitated networks. When the numbers are probabilities, one can study the structure of transitions between states of social and ecological systems using the model of a markov chain.

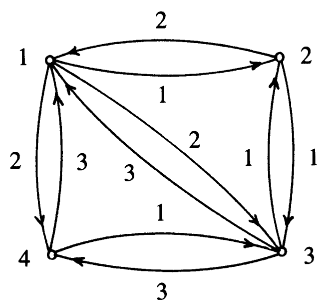

Fig. 1.6. A network N from the graph G in Fig. 1.1.

Graphs can represent algebraic groups as well as social groups. Although group theory has most often been applied to marriage systems, it is no less fundamental in the analysis of other forms of symbolism based on permutation relations. An example is provided by Dell Hymes (1971), who uses an informal group model to elucidate a basic genre, or semantic field, in Clackamas Chinook mythology. The genre is defined by the permutations resulting from (1) the upholding versus the violation of a social norm and (2) an adequate versus an inadequate response to an empirical situation.

These two relations yield four types, each of which has its own characteristic outcome.

Thus, for example, in the myth ''Kušaydi,'' the murderous hero eats something about which he has been warned and dies as a result. In the myth ''Seal took them to the ocean,'' the hero mistreats his elder brother but manages to survive many physical contests underwater by heeding Seal's advice. The outcome, however, is mixed; no accession of power or wealth, despite all the underwater accomplishments. In the myth ''Seal woman and her younger brother dwelt there,'' the heroine insists on maintaining social propriety at the expense of ignoring a danger, with tragic consequences. In ''Black Bear and Grizzly Woman and their sons,'' the heroes behave respectably and succeed in avenging their mother and outwitting Grizzly Woman.

Hymes characterizes his procedure by quoting Lévi-Strauss's celebrated definition of structuralist method enunciated in *Totemism* (1963b:16):

1. define the phenomenon under study as a relation between two or more terms, real or supposed;
2. construct a table of possible permutations between these terms;
3. take this table as the general object of analysis. . . .

Hymes's model can be clearly represented by a cube graph like that in Fig. 1.7, where *U* and *V* signify the upholding versus the violation of a social norm, and *A* and *I* signify an adequate versus an inadequate response to an empirical situation. In such a graph, each permutation corresponds to a distinctive combination of the numbers 0 and 1 at the corners. This is a model of the famous Klein group evoked in the structuralist analysis of myth (Lévi-Strauss 1968, 1981). It is also a specific instance of the general model of a boolean group (Hage and Harary 1983).

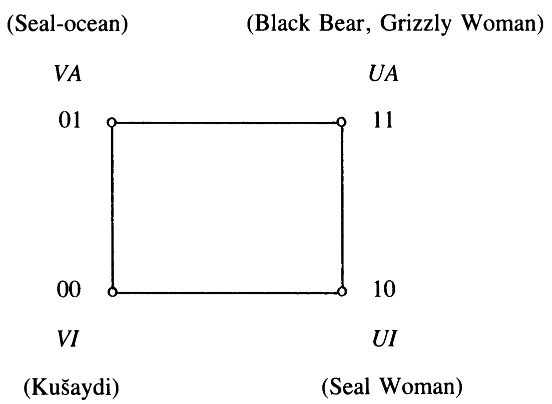

Fig. 1.7. A permutation group of Chinook myths.

Group models are basic to the analysis of symmetry, a property that is not limited to physical structures, but is also a characteristic of certain forms of primitive classification. Indeed, from Piaget's point of view, the axioms that define a group are fundamental operations in the ontogenesis of intelligence. The proper evaluation of his theories and of his phylogenetic extrapolations to primitive thought, which have recently become a topic of controversy in anthropology, depends on knowing what these axioms are.

Advantages of graphical models

Graph theory offers four advantages as a structural model in anthropology. First of all, graphical models are in some sense iconic; they look like what they represent and thus, unlike algebraic models, they are "linked with reality" (Berge 1962). It is easy to understand a social or cognitive structure as a graph open to inspection and amenable to manipulation for the elucidation of its structural properties.

Second, graph theory provides exact and rich definitions of such concepts as connectivity, transformation, duality, and centrality. This language serves both to clarify common anthropological metaphors and to extend structural analysis. Thus one can sort out what is meant or what should be meant or even what could be meant by terms like "connectivity" in social networks or "permutations" in symbolic systems. At the same time, one can conceptualize completely new topics, such as "simplicity" and "complexity," in either type of structure.

Third, graph theory contains techniques for the calculation of quantitative aspects of structure – for example, reachability in a communication network, distance in a trade network, or transitivity in a status system. This is facilitated by the representation of a graph as a matrix and hence the application of simple and easily programmed algebraic manipulations. Matrices also have ethnographic advantages. As Marriott (1968) discovered in his analysis of caste relations in India, they have a certain naturalness and inevitability in the representation of complex structures.

Finally, graph theory contains theorems. By stating as a hypothesis those properties of graphs that necessarily satisfy given empirical conditions, a theorem allows us to draw conclusions about certain properties of a structure from knowledge about other properties. A simple but historically interesting example taken from Ascher and Ascher's (1981) discussion of implicit or folk mathematics will serve to illustrate.

As the Aschers observe, "mathematics arises out of and is directly concerned with the domain of thought involving the concepts of number, spatial configuration and logic." These are concerns not only of professional mathematicians, but of numerous other professional groups and in fact of individuals in all cultures. One of their examples shows how a problem studied by Melanesian artists is the same one that dates the origin of graph theory in Western mathematics. They cite an early study of New Hebrides geometrical art in which A. Bernard Deacon (1934) describes a number of figures, some of which have religious, mythological, or ritual significance, and all of which have distinctive names. The figures are drawn in the earth around a framework of lines or dots. "In theory the whole should be done in a single continuous line which ends where it began; the finger should never be lifted from the ground, nor should any part of the line be traversed twice" (Deacon 1934:133). One of the simpler such drawings is "The wild cabbage," shown in Fig. 1.8.[3]

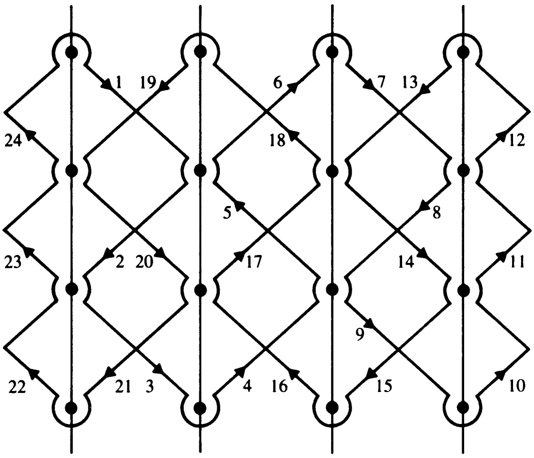

Fig. 1.8. A New Hebrides geometrical drawing (from Deacon 1934).

It is interesting to compare this drawing to the one in Fig. 1.9(a), which depicts the famous seven bridges of Königsberg and dates the origin of graph theory with the Swiss mathematician Leonhard Euler in 1736.[4] Euler (1707–1782) began the formal study of graph theory (as well as topology, which was earlier called *analysis situs*) when he settled in the negative a famous unsolved puzzle of his day called the "Königsberg Bridge Problem." As Euler himself said:

> The problem, which I am told is widely known, is as follows: in Königsberg in Prussia, there is an island A, called *the Kneiphof*; the river which surrounds it is divided into two branches, as can be seen in Fig. [1.9(a)], and these branches are

[3]The cover design, which features one of these drawings, owes much to the Russian painter Andrea Morguloff.

[4]We note that Fig. 1.9(a) is taken from Euler's original paper, as reproduced in the book on the history of graph theory by Biggs, Lloyd, and Wilson (1976), and unlike many depictions, shows the bridges as they really were.