

Cambridge University Press

978-0-521-27290-2 - Algebra Through Practice: A Collection of Problems in Algebra with Solutions, Groups

T. S. Blyth and E. F. Robertson

Frontmatter

[More information](#)

---

Algebra through practice

**Book 5: Groups**

Cambridge University Press

978-0-521-27290-2 - Algebra Through Practice: A Collection of Problems in Algebra with Solutions, Groups

T. S. Blyth and E. F. Robertson

Frontmatter

[More information](#)

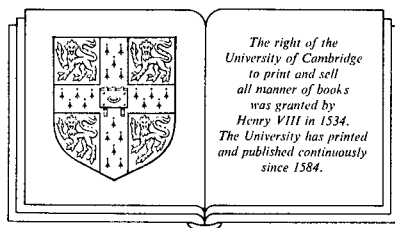
# Algebra through practice

*A collection of problems in algebra with solutions*

## Book 5 Groups

T. S. BLYTH ◦ E. F. ROBERTSON

*University of St Andrews*



CAMBRIDGE UNIVERSITY PRESS

*Cambridge*

*London New York New Rochelle*

*Melbourne Sydney*

Cambridge University Press

978-0-521-27290-2 - Algebra Through Practice: A Collection of Problems in Algebra with Solutions, Groups

T. S. Blyth and E. F. Robertson

Frontmatter

[More information](#)

---

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521272902](http://www.cambridge.org/9780521272902)

© Cambridge University Press 1985

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 1985

Re-issued in this digitally printed version 2008

*A catalogue record for this publication is available from the British Library*

*Library of Congress Catalogue Card Number: 83-24013*

ISBN 978-0-521-27290-2 paperback

Cambridge University Press

978-0-521-27290-2 - Algebra Through Practice: A Collection of Problems in Algebra with Solutions, Groups

T. S. Blyth and E. F. Robertson

Frontmatter

[More information](#)

---

# Contents

Preface	vii
Background reference material	viii
<b>1: Subgroups</b>	<b>1</b>
<b>2: Automorphisms and Sylow theory</b>	<b>10</b>
<b>3: Series</b>	<b>15</b>
<b>4: Presentations</b>	<b>22</b>
<b>Solutions to Chapter 1</b>	<b>29</b>
<b>Solutions to Chapter 2</b>	<b>51</b>
<b>Solutions to Chapter 3</b>	<b>62</b>
<b>Solutions to Chapter 4</b>	<b>79</b>
<b>Test paper 1</b>	<b>94</b>
<b>Test paper 2</b>	<b>96</b>
<b>Test paper 3</b>	<b>98</b>
<b>Test paper 4</b>	<b>100</b>

Cambridge University Press

978-0-521-27290-2 - Algebra Through Practice: A Collection of Problems in Algebra with Solutions, Groups

T. S. Blyth and E. F. Robertson

Frontmatter

[More information](#)

## Preface

The aim of this series of problem-solvers is to provide a selection of worked examples in algebra designed to supplement undergraduate algebra courses. We have attempted, mainly with the average student in mind, to produce a varied selection of exercises while incorporating a few of a more challenging nature. Although complete solutions are included, it is intended that these should be consulted by readers only after they have attempted the questions. In this way, it is hoped that the student will gain confidence in his or her approach to the art of problem-solving which, after all, is what mathematics is all about.

The problems, although arranged in chapters, have not been 'graded' within each chapter so that, if readers cannot do problem  $n$  this should not discourage them from attempting problem  $n + 1$ . A great many of the ideas involved in these problems have been used in examination papers of one sort or another. Some test papers (without solutions) are included at the end of each book; these contain questions based on the topics covered.

TSB, EFR  
St Andrews

Cambridge University Press

978-0-521-27290-2 - Algebra Through Practice: A Collection of Problems in Algebra with Solutions, Groups

T. S. Blyth and E. F. Robertson

Frontmatter

[More information](#)

## Background reference material

Courses on abstract algebra can be very different in style and content. Likewise, textbooks recommended for these courses can vary enormously, not only in notation and exposition but also in their level of sophistication. Here is a list of some major texts that are widely used and to which the reader may refer for background material. The subject matter of these texts covers all six of the present volumes, and in some cases a great deal more. For the convenience of the reader there is given overleaf an indication of which parts of which of these texts are most relevant to the appropriate sections of this volume.

- [1] I. T. Adamson, *Introduction to Field Theory*, Cambridge University Press, 1982.
- [2] F. Ayres, Jr, *Modern Algebra*, Schaum's Outline Series, McGraw-Hill, 1965.
- [3] D. Burton, *A first course in rings and ideals*, Addison-Wesley, 1970.
- [4] P. M. Cohn, *Algebra* Vol. I, Wiley, 1982.
- [5] D. T. Finkbeiner II, *Introduction to Matrices and Linear Transformations*, Freeman, 1978.
- [6] R. Godement, *Algebra*, Kershaw, 1983.
- [7] J. A. Green, *Sets and Groups*, Routledge and Kegan Paul, 1965.
- [8] I. N. Herstein, *Topics in Algebra*, Wiley, 1977.
- [9] K. Hoffman and R. Kunze, *Linear Algebra*, Prentice Hall, 1971.
- [10] S. Lang, *Introduction to Linear Algebra*, Addison-Wesley, 1970.
- [11] S. Lipschutz, *Linear Algebra*, Schaum's Outline Series, McGraw-Hill, 1974.

Cambridge University Press

978-0-521-27290-2 - Algebra Through Practice: A Collection of Problems in Algebra with Solutions, Groups

T. S. Blyth and E. F. Robertson

Frontmatter

[More information](#)

- [12] I. D. Macdonald, *The Theory of Groups*, Oxford University Press, 1968.
- [13] S. MacLane and G. Birkhoff, *Algebra*, Macmillan, 1968.
- [14] N. H. McCoy, *Introduction to Modern Algebra*, Allyn and Bacon, 1975.
- [15] J. J. Rotman, *The Theory of Groups: An Introduction*, Allyn and Bacon, 1973.
- [16] I. Stewart, *Galois Theory*, Chapman and Hall, 1975.
- [17] I. Stewart and D. Tall, *The Foundations of Mathematics*, Oxford University Press, 1977.

### References useful for Book 5

- 1: Subgroups [4, Sections 9.1, 9.6], [6, Chapter 7], [8, Sections 2.1, 2.11], [12, Chapters 1–6], [13, Sections 13.1, 13.4], [15 Chapters 1–4].
- 2: Automorphisms and Sylow theory [4, Sections 9.4, 9.8], [8, Section 2.12], [12, Chapter 7], [13, Section 13.5], [15, Chapter 5].
- 3: Series [4, Sections 9.2, 9.5], [12, Chapters 9,10], [13, Sections 13.6–13.8], [15, Chapter 6].
- 4: Presentations [4, Section 9.9], [12, Chapter 8], [15, Chapter 11].

In [8] morphisms are written on the left but permutations are written as mappings on the right. In [4] and [12] all mappings (including permutations) are written as mappings on the right. In American texts ‘solvable’ is used where we have used ‘soluble’.