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London Mathematical Society Lecture Note Series. 81

Skew Fields

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CAMBRIDGE UNIVERSITY PRESS
Cambridge
London New York New Rochelle
Melbourne Sydney

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521272742

First published 1983
Re-issued in this digitally printed version 2007

A catalogue record for this publication is available from the British Library

Library of Congress Catalogue Card Number: 82-22036

ISBN 978-0-521-27274-2 paperback

Max Deuring

zum 75. Geburtstag

am 9. Dezember 1982

gewidmet

CONTENTS

PREFACE

This is a substantially extended version of the notes on 25 lectures delivered at the Pennsylvania State University during Spring Term 1981 under 572.2 ("Special Topics in Algebra").

Most of the material has been presented earlier elsewhere: in the Seminar Bielefeld-Göttingen during the Summer Term 1967 (mainly Part III), in lectures at the Universität Bielefeld during the Academic Year 1977/78 and in lectures at the Université de Grenoble in February/March 1979 (mainly §§19/20.). Some of the material has been discussed afterwards in the course of different lectures which I delivered at the Universität Bielefeld during the Academic Year 1981/82.

The text falls into three parts: Skew fields and simple rings, Skew fields and Brauer groups, and Reduced K_1-theory of skew fields. As regards their contents the reader is advised to consult the introductory remarks at the beginning of each of these parts on pages 1, 57 and 125 respectively.

During the preparation of the final draft of these notes I have enjoyed the assistance of B. Fein (Oregon State University), D. Garbe (Universität Bielefeld), I.M. James (Oxford University, editor of the LMS Lecture Note Series), Ch. Preston (Universität Bielefeld), S. Rosset (Tel-Aviv University), B. Weisfeiler (Pennsylvania State University) and J. Tate (Harvard University, who communicated so far unpublished work to me and gave permission to publish his arguments here; cf. §11.); I am grateful for their help.

Moreover, I want to take this opportunity to express gratitude to G. Andrews, D. Brownawell, D. James, D. Rung, L.N. Vaserstein and B. Weisfeiler who enabled my visit to the Pennsylvania State University (during the Academic Year 1980/81) or made the stay there so enjoyable for me and my family.

Finally I must report that I have greatly benefitted from the help of P.M. Cohn (Bedford College, University of London) whose assistance and encouragement only made it possible for this book to appear. I am most grateful for all his help.

Bielefeld
August 1982

CONVENTIONS ON TERMINOLOGY

As usual, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$ stand for the natural numbers, integers, rational numbers, real numbers and complex numbers respectively. The French reader should note $0 \notin \mathbb{N}$.

We write $X := Y$ if X is defined by Y.

From group theory we adopt the following (standard) notation:

$H \leq G$	H is a subgroup of G,
$H \trianglelefteq G$	H is a normal subgroup of G,
$[G,G]$	commutator subgroup of G,
$G^{ab} := G/[G,G]$	commutator factor group of G,
$Z(G)$	centre of G.

If R is a ring, then we usually assume that it has a unit element, denoted by 1 (not necessarily $1 \neq 0$; note that $1 = 0$ implies $R = \{0\}$), which is inherited by subrings, preserved by homomorphisms and acts unitally on all R-modules. Moreover, we use the notation:

R^+	additive group of R,
R^*	multiplicative group of R (if $1 \neq 0$),
$Z(R)$	centre of R,
$M_n(R)$	ring of (n,n)-matrices over R,
$GL_n(R) := M_n(R)^*$	general linear group over R.

We call a ring D a *skew field* if $D^* \cup \{0\} = D$. We assume the reader to be familiar with the fact that left/right vector spaces over a skew field are always free of unique rank (called the *left/right dimension*); the proofs of these facts work precisely as in the commutative case (known from Linear Algebra).

Finally we point out that we assume a good knowledge of (ordinary) Field Theory (including Galois Theory of finite field extensions); here *field* stands for commutative field and $\mathbb{F}_q$ stands for the (finite) field with q elements.