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Excerpt
More information

1 Introduction

Nuclear physics is an intriguing subject because of the great variety of phenomena that occur. Nuclei exhibit many different types of behaviour, from the classical, where the nucleus behaves like a liquid drop, to the quantum-mechanical, where nuclei show a shell structure similar to that found in atoms. It is a considerable intellectual challenge to try and understand this behaviour and many models have been devised. These models are described in chapter 2 and used to understand the principal properties of nuclei and their excited states.

The study of beta and gamma decay in nuclei has given considerable information on the structure of nuclei; and from experiments on beta decay on the nature of the weak interaction, for example parity violation and the helicity of the neutrino. These topics are discussed in chapters 3 and 4.

The importance of quantum-mechanical tunnelling in nuclear physics is illustrated by a discussion in chaper 5 of α-decay, fission and thermonuclear fusion. The last two have considerable significance in other fields, for example nuclear power and nuclear astrophysics.

The interactions between nuclei offer a rich variety of phenomena and these are described in chapter 6. The basic types of nuclear reactions: compound nucleus, direct and deep-inelastic, and typical features such as the occurrence of resonances and characteristic angular distributions, are first described before reaction theories are developed.

The forces between nucleons are discussed in chapter 7. The information from neutron–proton and proton–proton scattering is analysed and the characteristics of the nuclear force explained. The connection between the two-nucleon force and the effective interaction between nucleons in nuclei is described and the reason for the success of the independent-particle model of the nucleus is discussed. Finally, the connections between different aspects of nuclear behaviour, in particular collective and single-particle, are drawn together in chapter 8.

In the remainder of the introduction there is a short historical review of nuclear physics followed by a description of the basic features and characteristic dimensions of nuclei. The general technique of scattering particles off nuclei to find out about their size and structure is then explained and examples are given. The spectra of excited states in nuclei with the same number of nucleons are found to have striking similarities which are a consequence of the charge independence of the nuclear force. This is formalised through the introduction of the concept of isospin, which provides an illustration of the important connection between invariance principles and conservation laws.

1.1 Historical review

In 1897 J. J. Thompson discovered the electron and found that most of the mass of an atom was positively charged. He proposed the idea that the atom was rather like a plum pudding with a uniform distribution of positive charge in which the negatively charged electrons were embedded. The previous year Becquerel had established that some atoms gave off ionising radiations, and three types of radioactivity (alpha, beta, gamma) were soon identified. In 1909 Rutherford and Royds identified the alpha rays as ionised helium atoms. Studies of the scattering of alpha particles as they pass through a thin foil led Rutherford to the realisation in 1911 that the positive charge of an atom was concentrated at the centre of the atom, in what is called the nucleus, with the negatively charged electrons surrounding it. From experiments, performed by Geiger and Marsden, on the alpha particle scattering off gold, Rutherford was able to conclude that the radii of the gold and alpha particle nuclei must be of the order of 10 fm (10^{-14} m).

Geiger and Marsden also showed that the positive charge of the nucleus was roughly half of its atomic weight relative to hydrogen. In 1911, Soddy conjectured from studies of radioactivity the existence of isotopes, which are atoms with different nuclear masses but with the same charge and number of atomic electrons and hence the same chemical behaviour. This led to the idea that the nucleus contained protons and electrons, the electrons neutralising some of the protons, but not appreciably altering the mass of the nucleus as the mass of an electron is only $\sim 1/2000$ of the proton's mass. This picture was also thought to account for the origin of electrons in beta radioactivity.

Progress on understanding the atom was made in 1913 when Bohr postulated that atoms only existed in discrete energy levels with the electrons moving around the nucleus with quantised amounts of angular momentum. This model was able to explain the spectrum of the hydrogen atom very well. It was also used by Moseley in 1913 to account for the dependence he had found of the energy of characteristic X-rays on the charge of the atom, which enabled elements to be identified by their characteristic X-rays.

The first nuclear reaction was seen by Rutherford in 1919 when nitrogen was irradiated by alpha particles and protons were produced. But after this, progress on understanding the structure of nuclei was slow, mainly because the only particles available to probe the nucleus were alpha particles from naturally radioactive materials. In the meantime, continued research on beta decay had indicated, by 1928, an apparent violation of energy conservation, which was only resolved by Pauli's hypothesis in 1930 that there was another particle, the neutrino, with which the electron shared the β-decay energy.

After the development of quantum mechanics in 1925 by Schrödinger and Heisenberg, the energetics of alpha decay were explained by Gamow and by Gurney and Condon in 1928 as arising through quantum-mechanical tunnelling of the alpha particle through the nuclear potential barrier. It was also realised that there were considerable difficulties with the proton and electron model of the nucleus. Using the uncertainty principle it was estimated that the energy of an electron within the nucleus was of the order of 50 MeV so it was unclear what was keeping the electron within the nucleus. Furthermore, by 1929 both the spin (integral) and statistics (Bose) of the ^{14}N nucleus were found to be inconsistent with the nucleus containing protons and electrons.

These problems were resolved by the discovery of the neutron by Chadwick in 1932, which led Heisenberg later that year to propose that nuclei consisted of just protons and neutrons (though he envisaged the neutron as made up of a proton and an electron). Such a model was consistent with the observed spin and statistics of ^{14}N and the idea of Iwanenko (1932), made quantitative by Fermi in 1934, that the electron was created in a β-decay, like a photon in a γ-decay, removed the need for electrons to exist in any form within nuclei. The neutron plus proton model of the nucleus was gradually accepted and the subject of modern nuclear physics really dates from this time (1932).

1.2 The scattering of particles by nuclei

The early experiments on α-particle scattering gave an indication of the size of nuclei and with beams of high-energy particles much more detailed information has been obtained. In quantum mechanics the scattering of a particle is described by Fermi's golden rule:

$$w = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f$$

where w is the scattering probability per unit time, M_{if} is the transition matrix element between initial and final states and ρ_f is the density of final states. This is a general relation and does not require the perturbation V causing the transition to be weak for its validity. However, if V is weak then M_{if} can be evaluated to a good approximation using first-order perturbation theory, also called the Born approximation. In this approximation M_{if} is given by the volume integral:

$$M_{if} = \int \psi_f^* V \psi_i \, d\tau$$

where ψ_i and ψ_f are the initial and final wavefunctions of the scattered particle. For $V = (g^2/4\pi r)e^{-\mu r}$, where g is a measure of the strength of the interaction and μ^{-1} of its range, and a scattering from momentum $\mathbf{p}_i = \hbar \mathbf{k}_i$ to $\mathbf{p}_f = \hbar \mathbf{k}_f$ then $\psi_i = L^{-3/2} \exp(i\mathbf{k}_i \cdot \mathbf{r})$, $\psi_f = L^{-3/2} \exp(i\mathbf{k}_f \cdot \mathbf{r})$ and:

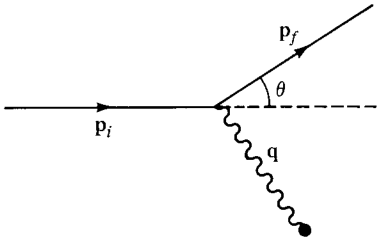
$$\begin{aligned} M_{if} &= \frac{g^2}{4\pi L^3} \int \exp(i\mathbf{q} \cdot \mathbf{r}) \frac{e^{-\mu r}}{r} \, d\tau \qquad (1.1) \\ &= \frac{g^2}{4\pi L^3} \int \exp(iqr\cos\theta - \mu r) r \sin\theta \, d\theta \, dr \, d\phi \\ &= \frac{g^2/L^3}{q^2 + \mu^2} \end{aligned}$$

where $\hbar \mathbf{q} = \mathbf{p}_i - \mathbf{p}_f$ is the momentum transfer in the scattering and L^3 is the normalisation volume.

The matrix element M_{if} is thus the Fourier transform of the potential. For a Coulomb potential $\mu = 0$ and $g = e/\sqrt{\varepsilon_0}$ and in the elastic scattering of a light charged particle by a heavy nucleus the momentum transfer $\mathbf{q}$ is given by $(\hbar q)^2 = 4p_0^2 \sin^2 \frac{1}{2}\theta$ (see figure 1.1) where $|\mathbf{p}_i| = |\mathbf{p}_f| = p_0$ so:

$$M_{if} \propto \frac{1}{p_0^2 \sin^2 \frac{1}{2}\theta} \qquad (1.2)$$

The scattering probability, which is proportional to $|M_{if}|^2$, is therefore proportional to $\sin^{-4} \frac{1}{2}\theta$, which is the angular dependence of the Rutherford scattering formula.



1.1. The relation between the momenta $\mathbf{p}_i$ and $\mathbf{p}_f$, and the momentum transfer $\mathbf{q}$.

1.2.1 Form factor

Generally the source of the perturbation has a spatial extent and this modifies the matrix element M_{if}. For example in the Coulomb scattering by a spherical charge distribution $Z\rho(\mathbf{R})$ the perturbation $V_\rho(\mathbf{r})$ is given by:

$$V_\rho(\mathbf{r}) = \int \rho(\mathbf{R})\, V(\mathbf{r}-\mathbf{R})\, \mathrm{d}^3R$$

where V is the Coulomb potential energy due to a point charge Z.

The matrix element M_{if} is given by the Fourier transform $\bar{V}_\rho(q^2)$ of $V_\rho(\mathbf{r})$. The perturbation $V_\rho(\mathbf{r})$ is a convolution of $\rho(\mathbf{R})$ and $V(\mathbf{r}-\mathbf{R})$ so by the convolution theorem:

$$\bar{V}_\rho(q^2) = F(q^2)\,\bar{V}(q^2)$$

where

$$\bar{V}(q^2) = \frac{1}{L^3}\int \exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r})\, V(\mathbf{r})\, \mathrm{d}^3r$$

and

$$F(q^2) = \int \exp(\mathrm{i}\mathbf{q}\cdot\mathbf{R})\rho(\mathbf{R})\, \mathrm{d}^3R \tag{1.3}$$

So the effect of an extended source is to modify the matrix element M_{if} by the factor $F(q^2)$, which is called the form factor. Note that $F(0)=1$ for all distributions. Expanding the exponential gives the following expression for $F(q^2)$:

$$F(q^2) = 1 - \frac{q^2\langle R^2\rangle}{6} + \cdots \tag{1.4}$$

so for values of q such that $q^2\langle R^2\rangle \ll 1$ then $F(q^2)=1$, which is the value for a point charge distribution. The above expression (1.3) for $F(q^2)$ is equivalent to:

$$F(q^2) = \int \rho(R)\,\frac{\sin qR}{qR}\, 4\pi R^2\, \mathrm{d}R \tag{1.5}$$

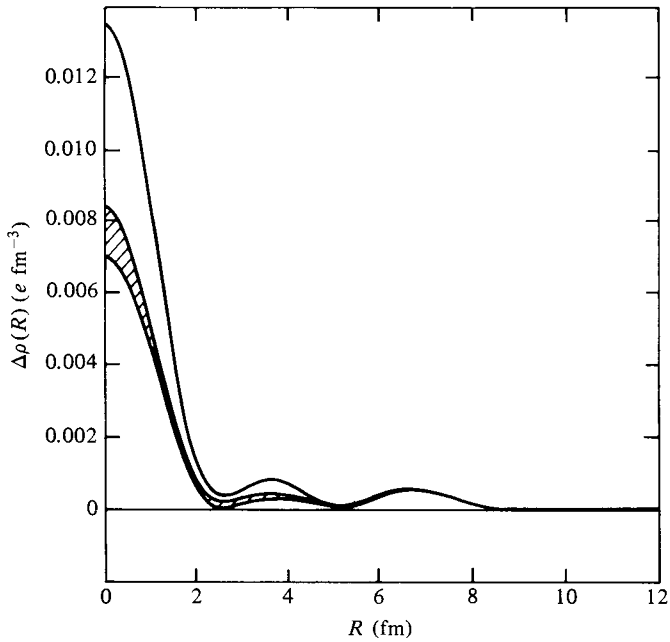
1.2.2 The scattering of electrons by nuclei

If the cross-section for electron scattering off a nucleus is measured over a wide range of momentum transfer q then the charge distribution can be measured from the inverse transform relation:

$$\rho(R) = \frac{1}{2\pi}\int F(q^2)\,\frac{\sin qR}{qR}\, 4\pi q^2\, \mathrm{d}q$$

This technique has been used to measure the charge distribution for many nuclei.

A particularly striking example of the information that can be obtained is seen in a comparison of the elastic electron scattering cross-section for ^{205}Tl and ^{206}Pb at an incident energy of 502 MeV. The charge density for each nucleus was deduced using the inverse transform relation together with information from measurements of muonic X-rays. The energies of these X-rays are sensitive to details of the nuclear charge distribution because of the much smaller size of the muon orbitals relative to the electron orbitals. (The radius is proportional to $1/m$ and since $m_\mu = 207 m_e$ their radii are much smaller.) The resulting charge-density difference is shown in figure 1.2. The shape determined by experiment is strikingly close to the one expected for a $3s_{1/2}$ wavefunction. In the simple shell model, using the notation $\pi \equiv$ proton and $\nu \equiv$ neutron, the ground state of ^{205}Tl is described as a pure $(\pi 3s_{1/2})^{-1}(\nu 3p_{1/2})^{-2}$ hole state in ^{208}Pb, i.e. two neutrons and one proton less than the doubly closed shell nucleus ^{208}Pb. The nucleus ^{206}Pb is described as a pure $(\nu 3p_{1/2})^{-2}$ hole state in ^{208}Pb. The difference is therefore one proton in a $3s_{1/2}$ state. The lack of exact agreement reflects the fact that the ground state wavefunctions are more complicated, and, if



1.2. Experimental charge-density difference between ^{206}Pb and ^{205}Tl together with the shape (solid line) expected for a $3s_{1/2}$ wavefunction. (From Cavedon, J. M. *et al.*, *Phys. Rev. Lett.* **49** (1982) 978.)

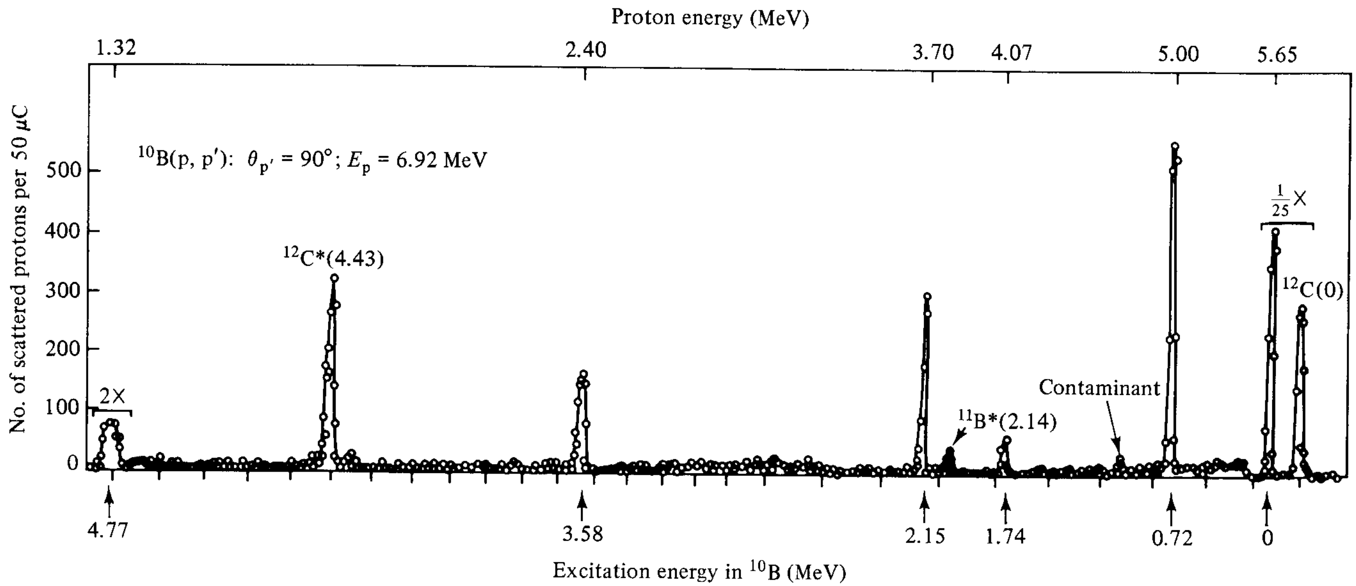
this is allowed for, much better agreement is obtained. The similarity in shape, though, provides clear evidence for independent-particle motion within a nucleus.

1.3 Nuclear spectra

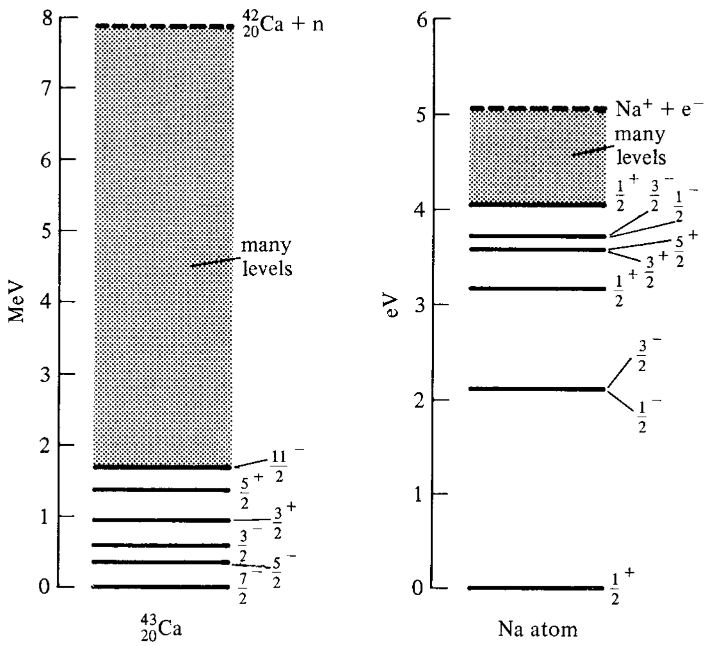
Besides giving detailed information on the size and charge distribution of nuclei, scattering experiments enable the excited states of nuclei to be studied. The basic technique is to bombard a thin foil, containing the nuclei under study, with a high-energy beam of particles (e.g. protons, ^{16}O nuclei) provided by an accelerator. The nuclei can be excited, just like atoms or molecules are excited when bombarded by electrons, and the scattered particles, or the gamma radiation following the de-excitation of the excited nuclei (or both), are detected. Their energies give information on the spectra of excited states (see figure 1.3).

The excited states arise when neutrons and protons are excited to higher quantum levels. A typical level scheme is shown in figure 1.4, with each state characterised by its excitation energy, its angular momentum (J) and parity (π). For comparison the excited atomic states of sodium are shown alongside. While a typical excitation energy in an atom is of the order of 1 eV, in nuclei it is of the order of 1 MeV. These magnitudes are what one would expect from applying the uncertainty principle to electrons confined in an atom (size: ~ 1 Å $= 10^5$ fm) and to nucleons (protons or neutrons) confined in a nucleus (size: ~ 10 fm). The natural units of energy and length in nuclear physics are MeV (1 MeV $= 10^6$ eV, 1 eV $= 1.6 \times 10^{-19}$ joules) and fermis (1 fm $= 10^{-15}$ metres). In these units $\hbar c = 197$ MeV fm ($= 1970$ eV Å), which is a useful number when estimating magnitudes in nuclear physics.

Also shown on the level scheme is the excitation energy when the nucleus is unbound to neutron emission, corresponding to the ionisation energy in an atom. Bound excited states generally decay by gamma decay, which is the emission of a photon, just as do atomic excited states. A typical lifetime for such an electromagnetic decay emitting a 1 MeV photon is $\sim 10^{-12}$ seconds in a nucleus, while in atoms for photons of 1 eV it is $\sim 10^{-8}$ seconds. If the excited state is unbound in a nucleus then particle decay usually occurs with a typical lifetime of 10^{-18} seconds. Such a short lifetime is equivalent to an energy width of approximately 1 keV. The connection between mean lifetime, τ, and energy width, Γ, is $\Gamma\tau = \hbar$. Noting that $c = 3 \times 10^{23}$ fm s^{-1} and $\hbar c \approx 200$ MeV fm gives $\Gamma = 0.7$ keV if $\tau = 10^{-18}$ seconds. Nuclear ground states which are beta radioactive have lifetimes which are very strongly dependent on the energy release in the decay but are always greater than 10^{-3} seconds. This much longer timescale



1.3. Spectrum of protons scattered from ^{10}B nuclei. In ^{10}B(p, p′) p refers to the incident and p′ to the scattered proton. Other peaks identified (* denotes an excited state) are from ^{12}C, ^{11}B and a contaminant. (Note the non-linear energy scale.) (Data from Bockelman, C. *et al.*, *Phys. Rev.* **92** (1953) 665.)



1.4. Diagram comparing the nuclear energy levels of ^{43}Ca with the atomic energy levels of Na. The ionisation energy of Na is ~5 eV and the neutron separation energy of ^{43}Ca is ~8 MeV.

for beta decay is caused by the decay taking place via the weak interaction.

Nucleons interact via the strong, electromagnetic and weak interactions which have characteristic strengths called coupling constants of $f = 1$ (strong), $\alpha = 1/137$ (electromagnetic) and $g \approx 10^{-6}$ (weak). It is now understood that the weak and electromagnetic interactions are part of an electroweak force while the strong force as it is known in nuclear physics is now realised to be a residual (but strong) effect of a more fundamental interaction between the quark constituents of a nucleon, rather like the Van der Waals force between molecules being a residual effect of the electrostatic interactions of the molecular constituents. In nuclear physics a good description of the strong nuclear force is obtained in terms of pion exchange, first proposed by Yukawa in 1935. Its range is short (~1.4 fm).

In the interaction between two nuclei a very important concept is that of cross-section (σ), which is the effective cross-sectional area that the pair of nuclei possess for a particular reaction process. For any interaction to take place the classical estimate of σ would be the sum of the cross-sectional areas of the two nuclei involved, which is of the order of $(10\,\text{fm})^2$ which equals $10^{-28}\,\text{m}^2$ and is called 1 barn. Actual reaction cross-sections vary over a huge range and can be much larger than the classical estimate, but one millibarn would be quite typical.

Levels in nuclei with the same number $(A = Z + N)$ of protons and neutrons, called isobaric nuclei, show striking similarities which are not a reflection of a particular nuclear model but of an equality of the nuclear force between two neutrons, two protons or a neutron and proton in the same space-spin state. This equality can be formalised by introducing the concept of isospin. This concept is a useful one in nuclear physics and to develop it fully one must realise the connection between invariance and conservation laws.

1.4 Invariance and conservation laws

To illustrate the connection between conservation laws and invariance principles consider a system described by a wavefunction ψ. Rotate the system by a small angle ε about the z-axis. Then the new wavefunction ψ' is related to the old one by:

$$\psi' = \psi + \varepsilon \frac{\partial \psi}{\partial \phi} + \cdots$$

$$\equiv \left(1 + \frac{\mathrm{i}\varepsilon}{\hbar} L_z\right) \psi \qquad \left(L_z \equiv -\mathrm{i}\hbar \frac{\partial}{\partial \phi}\right)$$

to first order in ε where L_z is the z component of the angular momentum operator.

If the energy of the system E after rotation by ε is the same as before then:

$$H\psi = E\psi \quad \text{and} \quad H\psi' = E\psi'$$

where H is the Hamiltonian of the system:

$$\therefore \quad \int \psi'^* H \psi' \, \mathrm{d}\tau = \int \psi^* H \psi \, \mathrm{d}\tau$$

$$\therefore \quad \int \psi^* \left(1 - \frac{\mathrm{i}\varepsilon}{\hbar} L_z\right) H \left(1 + \frac{\mathrm{i}\varepsilon}{\hbar} L_z\right) \psi \, \mathrm{d}\tau = \int \psi^* H \psi \, \mathrm{d}\tau \quad (L_z \text{ is Hermitian})$$

$$\therefore \quad \int \psi^* (H L_z - L_z H) \psi \, \mathrm{d}\tau = 0$$

$$\therefore \quad [H L_z] = 0$$

i.e. H must commute with L_z. Heisenberg's equation of motion for L_z is:

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle L_z \rangle = \frac{\mathrm{i}}{\hbar} \langle [H L_z] \rangle$$