

Contents

Preface	v
Notation and abbreviations	vii
1 Finite trials	1
1.1 Probability spaces	1
1.2 Real random variables and random vectors	3
1.3 Mixing, direct composition, and tree composition	13
1.4 Conditional probabilities	24
1.5 Independence	27
1.6 Independent random variables	32
1.7 The law of large numbers	35
2 Probability measures	38
2.1 General trials and probability measures	38
2.2 The extension theorem of probability measures	46
2.3 Direct products of probability measures	53
2.4 Standard probability spaces	60
2.5 One-dimensional distributions	67
2.6 Characteristic functions	80
2.7 The weak topology in the distributions	97
2.8 d -Dimensional distributions	102
2.9 Infinite-dimensional distributions	105
3 Fundamental concepts in probability theory	110
3.1 Separable perfect probability measures	110
3.2 Events and random variables	114
3.3 Decompositions and σ -algebras	123
3.4 Independence	129
3.5 Conditional probability measures	136
3.6 Properties of conditional probability measures	145
3.7 Real random variables	148
3.8 Conditional mean operators	157

<i>Contents</i>	<i>iv</i>
4 Sums of independent random variables	165
4.1 General remarks	165
4.2 Convergent series of independent random variables	170
4.3 Central values and dispersions	175
4.4 Divergent series of independent random variables	184
4.5 Strong law of large numbers	186
4.6 Central limit theorems	191
4.7 The law of iterated logarithms	199
4.8 Gauss's theory of errors	206
4.9 Poisson's law of rare events	209
Index	212