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978-0-521-26923-0 - Results and Problems in Combinatorial Geometry
V. G. Boltjansky and I. Ts. Gohberg
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Results and Problems in Combinatorial Geometry

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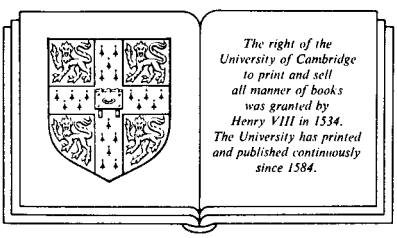

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FOREWORD

There are many elegant results in the theory of convex bodies that may be fully understood by high school students, and at the same time be of interest to expert mathematicians. The aim of this book is to present some of these results. We shall discuss combinatorial problems of the theory of convex bodies, mainly connected with the partition of a set into smaller parts.

The theorems and problems in the book are fairly recent; the oldest of them is just over thirty years old, and many of the theorems are still in their infancy. They were published in professional mathematical journals during the last five years.

We consider the main part of the book to be suitable for high school students interested in mathematics. The material indicated as complicated may be skipped by them. The most straightforward sections concern plane sets: §§1–3, 7–10, 12–14. The remaining sections relate to spatial (and even n-dimensional) sets. For the keen and well-prepared reader, at the end of the book will be found notes, as well as a list of journals, papers and books. References to the notes are given in round brackets, ( ), and references to the bibliography in square brackets, [ ]. In several places, especially in the notes, the discussion is at the level of scientific papers. We did not consider it inappropriate to include such material in a non-specialized book. We feel that it is possible to popularize science, not only for the layman but also for the benefit of the expert.

The book brings the reader up-to-date as far as the problems considered here are concerned. At the end of the book (§19) some unsolved problems are stated. Several of them are so intuitive and

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so easy to state that even able high school students can speculate about their solutions.

In conclusion, let us say a few words about combinatorial geometry itself. This is a new branch of geometry which is not yet in its final form; it is too early to speak of combinatorial geometry as a subject apart. Apart from the problems presented in this book, a group of problems connected with Helly's theorem (see Chapter 2 [37]) are without doubt related to combinatorial geometry, as are problems about packings and coverings of sets (see the excellent book by Fejes Tóth [12]), as well as a series of other problems. For the interested reader, we also very much recommend the book by Hadwiger and Debrunner [24], devoted to problems of combinatorial geometry of the plane, and the most interesting paper of Grünbaum [18], closely connected with the material presented to the reader.

The authors would like to take this opportunity to express their sincere gratitude to I. M. Yaglom, whose enthusiasm and friendly participation greatly contributed to improving the text of this book.

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INTRODUCTION TO THE ENGLISH TRANSLATION

This book originally appeared in Russian almost twenty years ago: nevertheless it is as fresh now as then. No better exposition of the main results has since appeared, and the problems stated at the end of the book still remain unsolved.

I would like to mention two books which appeared after this volume and which are closely related to this material. The first is "The Decomposition of Figures into Smaller Parts" by the same authors, which appeared in English translation in the University of Chicago Press in 1980, and also the book of V.G. Boltyansky and P.S. Soltan "Combinatorial Geometry of Different Classes of Convex Sets" Stiintsa, Kishinev, 1978 (in Russian). The first book is a popular book devoted only to combinatorial problems of the plane, and the second book is on the level of mathematical research monographs.

Finally, I would like to thank Cambridge University Press and Dr. David Tranah for their interest and cooperation.

I. Gohberg

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