Dual Models

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to Gilbert and Hugo
in grateful appreciation

Euclid alone has looked on beauty bare.

Edna St. Vincent Millay

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Foreword

Mathematics is a remarkable subject. Quite simply, it is enormous fun. Usually in mathematics one is manipulating symbols on a page, or maybe these days on a computer screen, and it can be hard to appreciate the inherent beauty of the ideas one is playing with. The subject thus acquires a reputation for difficulty.

Occasionally, though, mathematics starts to describe clearly visible things, and its beauty becomes accessible to everybody. In this book, Father Magnus Wenninger takes us along one of these accessible branches of mathematics. One only has to read his work, or better still, meet the author himself, to realise that Magnus Wenninger is an enthusiast. His life's hobby is polyhedra – three-dimensional shapes governed by precise formal rules of construction. Two-dimensional shapes have their interest, but we are bombarded all our lives with two-dimensional pictures and symbols, and such patterns have lost some degree of novelty. Four-dimensional shapes are definitely difficult to visualise and appreciate.

Three dimensions make a happy compromise. A three-dimensional shape can be constructed as an actual model (though sometimes it takes some effort to do so!) and held in one's hand. Myself, I first became seriously interested in polyhedra for just this reason. One of my teachers at Cambridge, the late Jeff Miller, had contributed academically to the subject and also had an exquisite little set of intricately folded paper models. These models were mostly of uniform polyhedra, whose construction details were first becoming widely known as a result of Magnus's first book, and I amused myself for quite some time making several of them and even making my local computer discover a special one.

But what is a ''uniform polyhedron'' or a ''regular polyhedron'' or a ''stellated polyhedron'' or any of the several other classifications? The answer is that each class of polyhedra obeys a certain set of rules, laid down in advance. For uniform polyhedra, the rules happen to be that all the vertices are equivalent and all the faces are regular polygons (equilateral triangles, squares, regular pentagons, or pentagrams, etc.).

Why, one may ask, use those particular rules? The answer to that is surprising and touches the very spirit of mathematics. We use these rules because we like what they produce. At the research frontier, mathematics is a free subject. It is more of a high art form than a science. We can do what we will in mathematics, inventing such rules as we wish and playing whatever games we like with them. Our mathematical training is then used as experience in finding which rules are productive of new results, which rules are contradictory (it is conventional to discard these), and which rules interlock with other rule-systems to give new insights into the whole subtle edifice of rules (axioms) and consequences (theorems).

Sometimes, as with arithmetic, the rules enable us to deal more effectively with natural objects and are so useful that we teach them even to young children. Sometimes, as with algebra, the rules enable us to deal with generalities of objects, and we teach them to older children. Sometimes the consequences of the rules are just pretty. So it is with polyhedra. Indeed, I know of no other branch of mathematics in which the link with aesthetics is so clear.

Now, what is a ''dual model''? Magnus will explain this to you much more clearly than I

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would. I shall merely remark that a dual is to its original as an octahedron is to a cube and invite you to collect your paper, scissors, and glue and read on. . .

John Skilling

March 1983
Cambridge, England

Preface

In the Epilogue of my book *Polyhedron models* I mentioned that none of the Archimedean duals had been presented and also that the stellation process described in that book for two of the regular polyhedra and for the two quasi-regular solids can be applied to any of the other Archimedean polyhedra, as well as to all their duals. In my book *Spherical models* I extended the techniques of model making to the modeling of spherical polyhedra, going thereby into a deeper presentation of the mathematical basis for polyhedral symmetry. This book, *Dual models*, now completes a significant body of knowledge with respect to polyhedral forms.

In this book I propose to follow the same style as that used in the two earlier ones, presenting models in photographs, along with line drawings, diagrams, and commentary. You will find here not simply a multiplication of geometric forms but an underlying mathematical theory that unifies and systematizes the whole set of duals of uniform polyhedra. Some of these models are not as complex as some of those in the first book. Also, the mathematical approach to geometrical forms used in the second book is brought into very practical application here. So I can assure you that the level of mathematics you will need in order to follow the details of drawing and calculation will remain at the high school or secondary level. You will need some knowledge of plane geometry and some acquaintance with geometrical constructions using ruler and compass, but the three-dimensional or solid-geometry aspects will be presented in the dress of two-dimensional or plane geometry. Furthermore, the recent easy availability of small electronic calculators has taken all the tedium out of paper-and-pencil calculations that at one time depended entirely on the slide rule or on the use of mathematical tables that involved squares and square roots and trigonometry along with logarithms. It seems to me that polyhedral shapes provide an ideal topic of investigation for secondary-level mathematics.

The convex Archimedean duals are well known from recent works now available. Some of these are given in the list of references at the end of this book. But very little, or nothing, so far as I know, has been published on the duals of nonconvex uniform polyhedra. The great polyhedronist Max Brückner was acquainted with many of these, as is evident from his work *Vielecke und Vielfläche*, published in 1900, but no recent book has taken up this topic with renewed interest. The purpose of this book is to fill this lacuna.

There exists a very close relationship between the stellation process and the generation of nonconvex uniform polyhedral duals. This relationship is presented in detail in this book. It is the underlying mathematical theory that unifies and systematizes the entire set of such duals. As such, it falls directly in line with the objective mentioned in the Epilogue of my first book: ''The object of an investigator would not be to multiply forms but to arrive at the underlying mathematical theory that unifies and systematizes whole sets of polyhedral forms.'' Furthermore, the stellation of convex Archimedean solids, and, even more, the stellation of their duals, can lead to some fantastically beautiful and interesting shapes, very useful for decorative purposes. Beauty really does not need to have uses, but the most frequent question that people ask when they see these shapes is ''What do you use them for?'' The aesthetic appeal that these forms have for the mind and the imagination should be enrichment enough. These shapes are interesting simply for the inherent relational aspects they have among themselves and with other simpler shapes. You will also see here that variations of strictly Archimedean forms will enter the picture, giving some hint of possibilities for the investigation of continuous transformations of geometric forms. This latter aspect, however, falls outside the scope of this book.

Special thanks must be extended to Gilbert Fleurent for the work he has done with his electronic printing calculator, from which he has obtained numerical data to produce marvelously accurate stellation patterns for the duals of the nonconvex snub polyhedra. Without his help I could never have finished this work. My thanks also to Hugo Verheyen for the interest he has taken in this work and for the suggestions he has made on how to include dual models of the so-called hemipolyhedra. I am indebted to H. Martyn Cundy for having sent to me an outline of the geometrical relationship between the stellation process and duality. This I have incorporated into the Introduction of this book. Norman W. Johnson, who was originally instrumental in ascribing names to all the uniform polyhedra, has again provided names for all the dual solids displayed in this book. Greek derivatives for the numbers and the shapes of the polyhedral parts continue to be used as heretofore, resulting in what most people find to be unpronounceably odd names. Because these names are so long and cumbersome to use, they will generally be omitted in referring to the models. The models will be designated by their numbers, the same numbers as those used in *Polyhedron models*. You will therefore find that book an indispensable companion volume to this one.

For the photography, I am indebted to Stanley Toogood, Andrew Aitken, John Dominik, and Hugh Witzmann. Excellent as the photography may be, it cannot do full justice to the models. Only when you handle a model yourself will you see the wonders that lie hidden in this world of geometrical beauty and symmetry.

M.J.W.