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 M. Crampin and F. A. E. Pirani  
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## **Applicable Differential Geometry**

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*To John Lighton Synge*  
*and to the memory of Alfred Schild.*

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## Preface

This book is based on lecture courses given by the authors over the past decade and a half to various student audiences, most of them final year undergraduates or beginning graduates. It is meant particularly for those who wish to study relativity theory or classical mechanics from a geometrical viewpoint. In each of these subjects one can go quite far without knowing about differentiable manifolds, and the arrangement of the book exploits this. The essential ideas are first introduced in the context of affine space; this is enough for special relativity and vectorial mechanics. Then manifolds are introduced and the essential ideas are suitably adapted; this makes it possible to go on to general relativity and canonical mechanics. The book ends with some chapters on bundles and connections which may be useful in the study of gauge fields and such matters. The “applicability” of the material appears in the choice of examples, and sometimes in the stating of conditions which may not always be the strongest ones and in the omission of some proofs which we feel add little to an understanding of results.

We have included a great many exercises. They range from straightforward verifications to substantial practical calculations. The end of an exercise is marked with the sign  $\square$ . Exercises are numbered consecutively in each chapter; where we make reference to an exercise, or indeed a section, in a chapter other than the current one we do so in the form “Exercise  $n$  of Chapter  $p$ ” or something of that kind. We conclude each chapter with a brief summary of its most important contents. We hope that beginners will find these summaries useful for review, while those for whom the subject is not entirely new will be able to tell from the summary whether a chapter treats topics they are already familiar with.

We have attempted to make the book as self-contained as is consistent with the level of knowledge assumed of its readers and with the practical limits on its length. We have therefore appended notes to several chapters; these notes summarise background material not covered in the text or give references to fuller treatments of topics not dealt with in detail. The notes should be the first recourse for a reader who meets something unfamiliar and unexplained in the main text.

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