

## Index of notation

$\mathbb{N}$	natural numbers
$\mathbb{N}_0$	$\mathbb{N} \cup \{0\}$
$\mathbb{Z}$	integers
$\mathbb{Q}$	rationals
$\mathbb{R}$	reals
$\mathbb{R}^n$	$n$ -dimensional Euclidean space
$\mathbb{R}_+^n$	$\{(x_1, \dots, x_n) \in \mathbb{R}^n \mid x_1 \geq 0\}$
$\mathbb{C}$	complex numbers
$n!$	$n$ factorial, $n \in \mathbb{N}$
$\binom{m}{n}$	binomial coefficient, " $m$ choose $n$ "
$(m, n)$	highest common factor of $m, n \in \mathbb{N}$
$[m/n]$	integer part of fraction $m/n$
$R[t]$	polynomials over a commutative ring $R$ with identity
$\phi_k(t)$	$k$ th cyclotomic polynomial
$\nabla(z)$	Conway potential function
$A(x)$	Alexander polynomial
$f * g$	resolvent of polynomials $f, g$
$GF(p)$	Galois field of $p$ elements, $p$ a prime
$RG$	group ring of $G$ over $R$
$GL(n, R)$	group of non-singular $n \times n$ matrices over ring $R$
$\det A$	determinant of matrix $A$
$SL(n, R)$	$\{A \in GL(n, R) \mid \det A = 1\}$
$\leq$	less than or equal to, is a subgroup of
$<$	less than, is a proper subgroup of
$\triangleleft$	is a normal subgroup of
$\dot{\cup}$	disjoint union of sets
$\setminus$	set-theoretic difference

$\emptyset$	empty set
$\delta_{ij}$	Kronecker delta
$\odot$	commutative face of diagram, copyright
$\longrightarrow$	mapping
$\dashrightarrow$	mapping whose existence is alleged
$\mapsto$	effect of mapping on an element
$\blacktriangleright$	injection, monomorphism
$\blacktriangleright\blacktriangleright$	surjection, epimorphism
$\cong$	isomorphism
$\exists!$	there is a unique
$1_X$	identity mapping on set X
inc	inclusion mapping
nat	natural homomorphism
Ker	kernel of a homomorphism
Im	image of a mapping, homomorphism
$\text{Hom}_G(A, B)$	the group of G-homomorphisms between G-modules A, B
$Z^n(G, A)$	n-dimensional cocycles from group G to G-module A
$B^n(G, A)$	n-dimensional coboundaries
$H^n(G, A)$	nth cohomology group of G with coefficients in A
$A \oplus B$	direct sum of A and B
$\Phi(A)$	intersection of maximal submodules of A
$d(A)$	minimal number of generators of A
$P_A$	$A/pA$
$A^G$	maximal G-trivial factor-module of A
$A_G$	maximal G-trivial submodule of A
e	identity element of G
E	trivial group
$\ell(w)$	length of reduced word w
$[x]$	order of $x \in G$ , modulus of $x \in C$
$x^y$	conjugate $y^{-1}xy$
$[x, y]$	commutator $x^{-1}y^{-1}xy$

$ X $	cardinality of set $X$ , underlying polyhedron of simplicial complex $X$	
$X^{-1}$	$\{x^{-1} \mid x \in X\}$ , $X \subseteq G$	
$X^{\pm 1}$	$X \cup X^{-1}$	
$\langle X \rangle$	subgroup of $G$ generated by $X \subseteq G$	
$\bar{R}$	normal closure of $R \subseteq G$	
$\langle X \mid R \rangle$	group on generators $X$ with relators $R$	
$F(X)$	free group on $X$	
$A(X)$	free abelian group on $X$	
$X^Y$	$\{x^y \mid x \in X, y \in Y\}$ , for $X, Y \subseteq G$	
$[X, Y]$	$\{[x, y] \mid x \in X, y \in Y\}$ , for $X, Y \subseteq G$ , the subgroup generated by this set, when $X, Y \leq G$	
$G'$	derived group of $G$	
$G^{\text{ab}}$	" $G$ abelianized", $G/G'$	
$Z(G)$	centre of $G$	
$\Phi(G)$	Frattini subgroup of $G$	
$G^p$	$\langle \{x^p \mid x \in G\} \rangle$ , $p$ a prime	
$M(G)$	Schur multiplier of $G$	
$\text{Aut } G$	automorphism group of $G$	
$\text{Inn } G$	group of inner automorphisms of $G$	
$\gamma_n(G)$	$n$ th term of lower central series of $G$	
$d(G)$	minimal number of generators of $G$	
$r(G)$	$d(G) + d(M(G))$	
$r'(G)$	minimal number of relations needed to define $G$	
$\text{def } G$	$\max\{ X  -  R  \mid \langle X \mid R \rangle \text{ a finite presentation of } G\}$	
$G_p$	the class of finite $p$ -groups for which $r(G) = r'(G)$	
$\dagger$	definition	} anotations in a coset enumeration table
$\ddagger$	bonus information	
$\S$	known information	
$\P$	coset collapse	

X+	adjunction of a generator	} Tietze transformations
X-	removal of a generator	
R+	adjunction of a relator	
R-	removal of a relator	
$ G:H $	index of subgroup $H \leq G$	
$G \times H$	direct product of groups, Cartesian product of sets	
$G * H$	free product of groups	
$G \wr H$	wreath product of groups	
$G \otimes H$	tensor product of groups	
$S_A$	group of all permutations of a set A	
$S_\infty$	$S_Z$	
$S_n$	symmetric group of degree n	
$Z_n$	cyclic group of order n	
$D_n$	dihedral group of degree n	
$Q_{2n}$	quaternionic group of order 4n	
$A_n$	alternating group of degree n	
$Z$	infinite cyclic group	
$B_n$	algebraical braid group on n strings	
$G_n$	geometrical braid group	
$U_n$	unpermuted braid group	
$G_n(w)$	cyclically presented group	
$A_n(w)$	$G_n(w)^{ab}$	
$F(r,n)$	Fibonacci group	
$F(r,n,k)$	generalized Fibonacci group	
$E(r,n)$	extended Fibonacci group	
$D(\ell,m,n)$	von Dyck group	
$\Delta(\ell,m,n)$	triangle group	
$Mac(a,b)$	Macdonald group	
$M(a,b,c)$	Mennicke group	
$W_\pm(a,b,c)$	Wamsley groups	

$d(P)$	degree of vertex $P$
$d(D)$	degree of face $D$
$M'$	boundary of map $M$
$\sum'$	sum over boundary vertices or faces
$T(k)$	} small cancellation conditions, $k \in \mathbb{N}$
$C(k)$	
$\chi$	Euler characteristic
$\partial$	topological boundary
$\#$	connected sum of surfaces
$\pi_1(X)$	fundamental group of path-connected space $X$
$H_1(X)$	$\pi_1(X)^{ab}$
$S^n$	$n$ -sphere
$T_n$	connected sum of $n$ tori, $n \in \mathbb{N}_0$
$P$	real projective plane
$P_n$	$T_n \# P$
$K_n$	$T_n \# P \# P$
$D$	disc
$M$	Möbius band
$C$	cylinder

## Index

- Abelian group 51
- abelianized group 51
- additive group of rationals 31
- Alexander polynomial 262
- algebraical braid group 276
- alternating group 50,117
- amphicheiral knot 261
- augmentation ideal 178,192
- augmentation mapping 178
- automorphism group 22,148,276
- Axiom of Choice 9,153,159
  
- Bar resolution 168
- barycentric subdivision 140
- base group 203
- basis 164
- Basis theorem 54,57,210,258
- Beyl 66,203
- bogus triangle 257
- braid 272
- braid group 37
- Brunner 76,244
- Burnside basis theorem 181,185
  
- Campbell 75
- cancellation triple 231
- Cartesian 34
- Cayley diagram 86,133,213
- Cayley's theorem 210
- central extension 189
- centralizer 23
- centre 281
- circulant matrix 77
- coboundary 155
- cocycle 154
- coefficients 170
- cohomology group 147,155,170
- Collins 230
- commutator 24
- commutator subgroup 51
- commutative diagram 3,156,188
- compact 246
- complement 8,152,195
- composition series 180
- concrete group 42
- conjugacy problem 39
- connected space 246
- connected sum 248
- continued fraction 285
- convex hull 246
- Conway potential function 268
- coset diagram 98,105
- coset collapse 90,102,120,125
- crossed homomorphism 171
- curvature 219
- cyclic group 24
- cyclically presented group 102
- cyclically reduced word 212
- cyclotomic polynomial 78
- cylinder 247

Cambridge University Press

978-0-521-23108-4 - Topics in the Theory of Group Presentations

D. L. Johnson

Index

[More information](#)

- Dedekind's theorem 53  
 deficiency 62,270  
 Dehn 39,266  
 Dehn's algorithm 218  
 derived group 51  
 dihedral group 43,46,69  
 Diophantine equation 118  
 direct product 24,32,160,203  
 disc 247  
 discriminant 200  
 double-dual 221  
 dual graph 132  
  
 Edge-path group 254  
 elementary knot deformation  
     259  
 embedding 258  
 empty word 5  
 Epstein 9  
 equivalence of extensions  
     153  
 essential singularity 236  
 Euler characteristic 250  
 Euler's formula 22,212,251,  
     271  
 exact sequence 162  
 exponent 201  
 exponent-sum 58  
  
 Factor set 155,189  
 false lovers' knot 272  
 Fibonacci group 31,60,75,89,  
     97,101,124,196,217,238  
 finitely presented group 23  
 Five-lemma 153  
 Fox 259  
  
 Fox derivative 264  
 Frattini subgroup 185,195  
 free abelian group 2,165  
 free group 1,229,279  
 free module 164  
 free presentation 23,182  
 free product 34,97,114,118,  
     197  
 Freiheitssatz 230  
 functorial properties 162,  
     170,174  
 fundamental group 245,254,  
     262,275  
  
 Gauss-Bonnet theorem 137  
 Gaussian integers 133  
 general linear group 43,50  
 generalized Coxeter group  
     116,118  
 generalized quaternion group  
     45  
 generator 23  
 geometrical braid group 276  
 Golod-Safarevič theorem 74,  
     147,198  
 Green 193  
 graph 21,212  
 group action 148  
 group extension 147  
 group of extensions 156  
 group of words 5  
 group ring 177  
  
 Hasse diagram 107,275  
 Hausdorff space 245  
 Havas 76

- homology group 258,266
- homotopy 253
- Hopfian group 22
  
- Icosahedron 136
- identification code 252,258
- identification space 248
- infinite dihedral group 197
- injective 9
- interesting group 63
- invariant factors 55
- inverse of o-knot 261
- inversion in a circle 143
- irreducible module 178
- isomorphism problem 39
  
- Jacobson radical 180
- Jordan curve theorem 213
  
- Klein bottle 247
- knot 258
- Knott 214
- knot group 37,262
- knot type 259
- Kostrikin 200
  
- Lexicographic ordering 10
- link 268
- Listing's knot 259,265
- localization 147
- locally Euclidean 246
- loop 133,215,253
- lower central series 201
- Lucas numbers 60
- Lyndon 238,244
  
- Macdonald 70
- Magnus 230
- manifold 245
- map 218
- Mennicke 70
- metacyclic group 63,84
- Miller 70
- minimal resolution 183,186
- minor 55
- Möbius strip 247
- Möbius transformation 287
- modular group 114,285
- module 147
- monoid 256
- Moser 86
- multiplication table 32
  
- Neighbourhood 245
- Nielsen transformation 19,281
- Nielsen-Schreier theorem 24, 60
- nilpotency class 73,201
- nilpotent group 73,202
- nilpotent ideal 180
- normal form 279
  
- Obverse of o-knot 261
- octahedron 136,231
- o-knot 260
- one-relator group 118,257
- orbit 179,185
- orientable surface 247
- orthogonal circles 144
  
- Path 253



- permutation group 185
- permutation representation
  - 96
- p-group 179
- piece 226
- plan 222
- planar graph 212
- planar projection 260
- pole 236
- polygonal knot 259
- presentation of group
  - extension 187
- pretzel 256
- prime knot 272
- primitive root of unity 45,78
- principal ideal domain 265
- projective 8
- projective plane 247
- proper power 280
- pure braid 275
  
- Quotient space 248
  
- Rank 2,57,164
- reduced van Kampen diagram
  - 216
- reduced word 5
- Reidemeister-Schreier
  - rewriting process 106,276
- relatively free 176
- relation matrix 53
- relator 23
- removable singularity 236
- resolution 168
- resolvent 82
- restriction mapping 164
  
- reverse of o-knot 261
- Richardson 76
- Robertson 75
- Roquette 147
- ruler and compass
  - construction 140
  
- San 29,51,276
- Schreier property 21
- Schreier transversal 10,25,
  - 85,105,277
- Schur Multiplier 60,173,
  - 203
- Seifert's method 265
- semi-direct product 151
- Serre 179
- short exact sequence 150,163
- simple group 85
- simplex 246
- simplicial complex 246
- skeleton 254
- small cancellation hypothesis
  - 212,226
- sphere 247
- split exact sequence 163
- split extension 63,151,279
- split link 271
- SQ-universal group 212
- stabilizer 185,274,287
- Sterling 76
- substitution test 29
- sum of knots 271
- surface 245
- surface group 253
- surgery 256
- Swan 60

Cambridge University Press

978-0-521-23108-4 - Topics in the Theory of Group Presentations

D. L. Johnson

Index

[More information](#)

- Sylow subgroup 206,210  
 symmetric group 21,46,60,  
     133,206,274  
 symmetrized relators 212
- Tame knot 259  
 tangle 269,283  
 tangle group 285  
 tensor product of groups 210  
 tessellation 131  
 tetrahedron 136  
 Tietze transformation 35,  
     49,60,89,128,196,237,255,  
     258,263,288  
 Todd-Coxeter coset enumeration  
     76,85  
 torsion free group 282  
 torsion element 212  
 torsion subgroup 117  
 torus 246  
 torus knot 270  
 trace 287  
 transversal 154  
 tree 254  
 trefoil knot 258,283  
 triangle group 114,130,196  
 triangulation 246
- trivial module 179  
 Trotter 261  
 true lovers' knot 272
- Underlying polyhedron 246  
 unknot 258  
 unpermuted braid group 274  
 upper central series 283
- Vandermonde determinant 77  
 van Kampen diagram 213  
 von Dyck group 35,50,61,71,  
     97,100,115,131,196,224  
 von Dyck's theorem 28
- Wamsley 66,70  
 Wedderburn's theorem 180  
 wild knot 259  
 Wirtinger presentation 262  
 Witt identity 40,71  
 word length 5  
 word problem 39,212,228,  
     244,279  
 wreath power 206  
 wreath product 203
- Zero-divisor 78