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978-0-521-22287-7 - Permutation Groups and Combinatorial Structures

N. L. Biggs and A. T. White

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Introduction

This book replaces an earlier one, entitled 'Finite Groups of Automorphisms' (No. 6 in the L. M. S. Lecture Note Series). When it became clear that the original book was still in demand, it was decided that a complete revision was preferable to a reprint. In this way we hoped to incorporate the fruits of experience, gleaned from readers' comments, and at the same time, to keep the book up-to-date by introducing some new material.

The entire book has been rewritten. In some sections of Chapters 1, 2 and 3 the earlier version has been followed quite closely, while in others (notably 1.5, 1.6, 2.4, 2.5, 3.4, 3.5), the material is treated differently. Chapter 4 (Groups and Graphs) is presented from a new viewpoint, beginning with the graphical representation of permutation groups. The algebraic theory of the adjacency matrix has been restricted to the case of strongly regular graphs, since the generalization to graphs of larger diameter is discussed elsewhere (reference [1] of Chapter 4). The material on the Higman-Sims group has been amplified. Chapter 5 (Maps) is completely new. We hope that its inclusion will lead to renewed interest in a subject where ideas from many different areas of mathematics come together.

We have tried to make the book suitable for use as a course text at advanced undergraduate or postgraduate level. For this reason, there are thirteen 'project' sections, which should provide a good test of a student's real understanding of the text. The topics treated in the project sections are occasionally needed in subsequent chapters. (See also the note below.)

We are grateful to our colleagues who, from January to March 1978, attended a weekly study group on these topics at Royal Holloway College; their support was much appreciated. Also, we received helpful comments and suggestions from several people, including P. J. Cameron,

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NOTE ON THE PROJECTS

The projects are not routine exercises. They are intended to be substantial pieces of work and, typically, a student might expect to spend several days working on each one. It might also be necessary for him to seek help from books, fellow-students, or his teacher. It is probable that no student will have time to attempt all the projects, but it would be an advantage to be familiar with the major results contained in them. Some parts of the projects are starred: these are either quite hard, or very time-consuming.