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Uniform Algebras and Jensen Measures

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Preface

These notes are based on lectures given in various courses and seminars over past years. The unifying theme is the notion of subharmonicity with respect to a uniform algebra. Dual to the generalized subharmonic functions are the Jensen measures.

Chapter 1 includes an abstract treatment of Jensen measures, which also includes the standard basic elements of Choquet theory. It is based on an approach of D.A.Edwards. Chapter 2 shows how the various classes of representing measures fit into the abstract setting, and Chapter 3 deals specifically with the algebra $R(K)$.

In Chapter 4, we present an example due to B.Cole of a Riemann surface R which fails to be dense in the maximal ideal space of $H^\infty(R)$.

Chapter 5 is based upon recent work of N.Sibony and the author concerning algebras generated by Hartogs series, and the abstract Dirichlet problem for function algebras. The abstract development is applied in Chapter 6 to algebras of analytic functions of several complex variables. Here the generalized subharmonic functions turn out to be closely related to the plurisubharmonic functions, and the abstract Dirichlet problem turns out to be Bremermann's generalized Dirichlet problem.

Chapters 7 and 8 are devoted to Cole's theory of the conjugation operator in the setting of uniform algebras. The problem is to determine which of the classical estimates relating a trigonometric polynomial and its conjugate extend to the abstract setting. Cole shows that many inequalities fail to extend to arbitrary representing measures, while

"all" inequalities extend to the context of Jensen measures.

In Chapter 9, the problem of characterizing the moduli of the functions in $H^2(\sigma)$ is considered. The discussion is based on Cole's proof of a theorem of Helson, which frees Helson's theorem from the underlying group structure.

References are given at the end of each chapter. At the very end of the notes, there is an index of symbols.

In preparing these notes, I have benefited from mathematical contacts with a number of people. Let me acknowledge first and foremost my debt to Brian Cole. His incisive ideas and remarkable results form the basis for a sizeable portion of these lecture notes. Special thanks go to Don Marshall, for writing up one of the preliminary versions of Chapter 9.

I would like to thank Julie Honig for her excellent work typing the penultimate version of the manuscript. And I would like to thank the staff at the Cambridge University Press for facilitating the publication of these notes.

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