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978-0-521-21975-4 - Continuous Crossed Products and Type III Von Neumann Algebras

A. van Daele

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# Continuous crossed products and type III von Neumann algebras

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Dedicated to Professor L. P. Bouckaert  
on the occasion of his seventieth birthday

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## Preface

These notes cover the material of a series of lectures given at the University of Newcastle upon Tyne on Takesaki's paper: 'Duality for crossed products and the structure of von Neumann algebras of type III' [16]. Since the appearance of Connes' thesis [2] and Takesaki's paper, the theory of crossed products has become very important in von Neumann algebras. An elementary and rather detailed treatment of the basics of this theory is given here, mainly intended for people who want an introduction to the subject. In part I, 'Crossed products of von Neumann algebras', I deal with general continuous crossed products. I introduce the notion in detail and give a proof of two important results. The first one is the commutation theorem for crossed products. It was obtained by Takesaki [16] in a special case, and by Digernes [4, 5] and Haagerup [8] in more general cases. The proof given here does not depend on the theory of dual weights, nor does it use any left Hilbert algebra. The second result given is Takesaki's duality theorem for crossed products with commutative groups.

In part II, 'The structure of type III von Neumann algebras', crossed products with modular actions are considered, that is those with the one-parameter group of  $*$ -automorphisms obtained by the Tomita-Takesaki theory, and I treat the structure theory of type III von Neumann algebras going with it [16]. Treatment is restricted to the case of  $\sigma$ -finite von Neumann algebras so that we can work with faithful normal states, and again our approach is different from the original one.

I would like to express my thanks to Professor J. Ringrose and to the other members of the department of pure mathematics of the University of Newcastle upon Tyne for their warm hospitality during my visit. This work was partially supported by the Science Research Council.

June 1976

A. Van Daele