The mathematical work of Charles Babbage

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CONTENTS

1

Introduction

Charles Babbage is an almost legendary figure of the Victorian era, yet relatively little is known about him. No authoritative account of his life and work has yet been published. In the absence of accurate knowledge, he is misrepresented as the eccentric genius, inventing computers which he never completed and quarrelling with almost everyone, especially the organ-grinders. Certainly Babbage was a man of highly individual talent, applying his ability with great success to a variety of subjects from economics to ballet, from deciphering to life insurance, from tool-making to astronomy. Indeed it is hard to think of any field of knowledge in which he did not excel, excepting possibly classics, for which he admitted a dislike. As far as his personal life is concerned, there is abundant, scarcely touched material available for studies of his exceptional personality, and when this is thoroughly examined it will almost certainly be discovered that he was a far different person from the one represented by popular misconception.

Primarily, Charles Babbage was a mathematician. In spite of the great variety of interests in other spheres, together with a considerable amount of family and social commitment, there is no doubt that he devoted himself essentially to a study of pure mathematics during the early years of his working life. In these years, as will be shown, his productive work was most original in content, exerting a strong influence on the course of British mathematics. It was recognised and esteemed by some of the greatest contemporary Continental mathematicians and contained many ideas, the value of which was not acknowledged till many years later.

At the age of thirty, Babbage embarked on his great life work:

the invention and manufacture of first the 'difference engine' and then the 'analytical engine'. This proved to be a task of such magnitude that it occupied him for the remaining fifty years of his life, but it was undoubtedly his major contribution to modern knowledge. During these years, he appeared to have abandoned his studies of pure mathematics, as far as published work was concerned. Nevertheless he always sought to provide a mathematical approach to each of the many and varied problems which he tackled. Whether discussing miracles, pin-making, postal services, geology, economics, politics or even his private life, he always attempted to formulate the problem in as mathematical a way as possible. It was his misfortune to encounter an age that was not so logical as himself. Mathematics was his first love, and it is largely because of his work that the world today, for better or worse, is much more mathematically aligned than it was a hundred years ago.

For these reasons I believe that it is not possible to assess the importance of Charles Babbage without an appreciation of his mathematical work. Hitherto, this has never been attempted. Most of his published mathematical work has not been read, let alone considered in relation to his subsequent life, and it is this defect which I propose to try to remedy. By attempting a critique of the whole of Babbage's mathematical methods and output, I hope to secure a necessary bridge-head from which the rest of his work can be properly analysed and appreciated.

Various accounts of Babbage's life and work are available. His own autobiography is *Passages from the Life of a Philosopher*, London, 1864. This is a most entertaining and interesting work. An unsystematic biography, it contains reminiscences and anecdotes, the historical accuracy of which cannot always be guaranteed. The reader receives a variety of information about the author, but learns nothing of such normally important biographical details as his place and date of birth, his wife and children.

The story of Babbage's life has recently been told in Maboth Moseley's work (the title of which indicates her general assessment of his character), *Irascible Genius*, London, 1964. This is the best biographical account of Babbage available, but contains little scientific appraisal.

There is also a long, unpublished and undated biography by Babbage's friend, Harry Wilmot Buxton, which is at present kept in a cowhide trunk containing several other relics of Babbage in

the Museum of the History of Science, Oxford. The full title of this work is

> Memoir of the life and labours of the late Charles Babbage Esq. F.R.S. formerly Lucasian Professor of Mathematics in the University of Cambridge. Comprising a description and historical account of his Analytical and Difference engines derived principally from his posthumous M.S.S. and papers by Harry Wilmot Buxton Esq., F.R.A.S.

The reference to the 'late Charles Babbage' in the title, and the fact that much of the text reads like an extended obituary notice, indicate that the work was compiled shortly after his death in 1871. It is written in a leisurely style and occupies over a thousand pages. The script is written on even-numbered pages only, with a corresponding blank odd-numbered sheet to each written one, intended presumably for correction and amendment. The author has made little use of these blank sheets, indicating that he did not make much progress in revising the text.

The book contains some biographical details of Babbage's early life which are probably more accurate than the ones from the *Passages from the Life of a Philosopher*. There is an account of his life at Cambridge, accompanied by a very long quotation from one of Babbage's unpublished works: 'The History of the Origin and Progress of the Calculus of Functions during the years 1809, 1810 . . . 1817.' As the title indicates, the bulk of the book is a very full account of the difference and analytical engines. It would appear that Buxton obtained his material for this subject from manuscripts that I found were still available when writing the eighth chapter of this book. There is also an interesting assessment of the work and character of Babbage, parts of which will be quoted in our final chapter. Although Buxton's tribute to Babbage is written in the most glowing terms, even he could have had little idea of Babbage's importance when viewed from a twentieth-century perspective.

Babbage produced volumes, some consisting entirely of diagrams, about his calculating engines; but relatively little was published. More detailed accounts of these machines were presented, with Babbage's full approval, by Dionysius Lardner on the difference engine, and by L. F. Menabrea and Lady Lovelace on the analytical engine. Most of the relevant articles, together with reports from the Royal Society and the British Association,

were collected and published by Babbage's son, Henry Prevost Babbage, in *Babbage's Calculating Engines*, London, 1889. There is also a more recent book containing a collection of most of the published accounts of the engines edited by P. and E. Morrison, *Charles Babbage and his Calculating Engines*, New York, 1961.

Amongst shorter articles on Babbage must be mentioned the entry in the *Dictionary of National Biography* by J. A. Fuller Maitland; L. H. Dudley Buxton's 'Charles Babbage and his difference engines', *Transactions of the Newcomen Society*, 1934, **14**, 43–65, based on his grandfather's unpublished book; and S. H. Hollingdale's 'Charles Babbage and Lady Lovelace – two 19th century mathematicians', *Bulletin of the Institute of Mathematics and its Applications*, 1966, **2**, 2–15.

Most modern books on computers contain a chapter of historical introduction in which acknowledgement to Babbage is made, the fullest of these accounts and the warmest of tributes appearing in B. V. Bowden's *Faster Than Thought*, London, 1953.

Babbage himself lists eighty items of his published work in an appendix to *Passages from the Life of a Philosopher* and at least eight of these are full-length books. There are, in addition, numerous unpublished works, including sixteen bulky volumes of correspondence in the British Museum Manuscripts Room. For the purposes of this book I have been particularly glad to discover two complete, unpublished mathematical works, the one in Oxford, already referred to, and 'The Philosophy of Analysis' in the British Museum Collection.

There has been some doubt as to when and where Charles Babbage was born. This arises mostly from his own indifference to the subject. H. W. Buxton gives 26 December 1791 as the date; L. H. D. Buxton 26 December 1792. The article in the *Dictionary of National Biography* states that he was born near Teignmouth on 26 December 1792, and P. and E. Morrison more cautiously suggest somewhere in Devonshire in 1792. M. Moseley quotes Babbage as saying on a certain occasion that he was born in London in 1792; she disagrees, stating categorically that he was born in Totnes on 26 December 1791.

The Times in its obituary notice of 23 October 1871 reported that 'Little is known of Mr. B's parentage and early youth except that he was born on the 26th December 1792.' A week later, the Rev. T. H. Hollier, a nephew of Babbage, wrote in to correct this statement to 26 December 1791.

The Marylebone Society tried unsuccessfully during the 1960s to resolve this question, in order to acquire the right information for a memorial plaque, but the solution was discovered by A. Hyman in 1975. He found a baptismal entry in the Parish Register of St Mary's, Newington, London: '6th January 1792 Charles s. of Benjamin and Betsy Plumleigh Babbage.'

Since 26 December seems to be the undisputed birthday, the correct year appears to be 1791, and Hyman suggests the most probable place of birth as the family residence of 44 Crosby Row, Walworth Road, very near to the Elephant & Castle, London.

I am indebted to H. W. Buxton for other details of Babbage's early life, and some of these are corroborated in *Passages from the Life of a Philosopher*. Charles was one of four children born to Benjamin Babbage, a member of the firm of Praed, Mackworth and Babbage (the London bankers), and Betsy Plumleigh Teape. His two brothers died in infancy, but his sister lived and survived him (*ibid.* p. 66).

He was a very weak and sickly child during his early years, and his parents were advised not to worry him with any kind of education. He grew stronger in adolescence, and, according to H. W. Buxton 'became finally exempt from those real or imaginary ills which had hitherto held him in thraldom' (*op. cit.*, p. 68). Babbage continues in *Passages from the Life of a Philosopher* (p. 10):

> Having suffered in health at the age of five years, and again at that of ten by violent fevers, from which I was with difficulty saved, I was sent into Devonshire and placed under the care of a clergyman (who kept a school at Alphington, near Exeter), with instructions to attend to my health; but, not to press too much knowledge upon me: a mission which he faithfully accomplished.

According to Buxton, he was then removed to the academy of the Rev. Stephen Freeman of Forty Hill, Enfield, Middlesex. This school consisted of thirty boys, one of Babbages's earliest friends being Frederick (later Captain) Marryat. His formal education began at this stage. He showed at once an indifference to the classics, but was strongly influenced by John Ward's *Young Mathematicians' Guide*, London, 1707.

After an unspecified number of years at Enfield, Babbage resided at home under the guidance of an Oxford tutor. In his now considerable spare time, he began to read as many mathematical books as he could find.

> Amongst these were Humphry Ditton's 'Fluxions', of which I
> could make nothing; Madame Agnesis' 'Analytical Institutions',
> from which I acquired some knowledge; Woodhouse's 'Princi-
> ples of Analytical Calculation', from which I learned the nota-
> tion of Leibnitz; and Lagrange's 'Theorie des Fonctions'. I
> possessed also the Fluxions of Maclaurin and of Simpson. (*Ibid.*
> p. 26.)

According to his unpublished work on the calculus of func-
tions, he began his researches into this subject at about this time,
late 1809.

He entered Trinity College, Cambridge, in October, 1810, but
migrated to Peterhouse where he took his B.A. in 1814, graduat-
ing as M.A. in 1817. His time at Cambridge was most fruitful
mathematically. He soon found that what he had learnt from
private study was more than that required for examination pur-
poses, and consequently he ignored the formal tuition. He
formed the Analytical Society together with J. F. W. Herschel,
George Peacock and others, for the conducting of mathematical
discussion and research. The work of this Society had a very
beneficial effect on British mathematics, which had severely
declined by the end of the eighteenth century.

At this time, Babbage and Herschel wrote the book *Memoirs of
the Analytical Society*, Cambridge, 1813, a work which indicated
the depth of their reading and the originality of their research.
Babbage also had his two long essays on the calculus of functions
published in the *Philosophical Transactions* of 1815 and 1816.

On leaving Cambridge in 1815, Babbage resided in London at
5, Devonshire Street. He was elected Fellow of the Royal Society
in 1816 and Fellow of the Royal Society of Edinburgh in 1820. He
helped to found the Royal Astronomical Society in 1820, being
Secretary for 1820–24 and later Vice-President, Foreign Secre-
tary and Member of the Council.

Little is known about his family life. He married Georgina
Whitmore in 1814 and they had eight children, four of whom
survived childhood. 1827 was a year of personal disaster in which
Babbage lost his father, his wife and two of his children, his own
health breaking down. He never married again.

In about 1821, be began work on his engines, a task which
occupied most of the rest of his working life, and involved him in
controversies with the Government and others. The details of this
part of Babbage's life are related in chapter 8.

Becoming increasingly critical about the state of scientific organisation in this country, he wrote a polemical book, *Reflections on the Decline of Science in England,* London, 1830, expressing great dissatisfaction with the Royal Society. He helped to found the British Association in 1831, being a Trustee from 1832–38. He also originated the Statistical Section of this Association at the Cambridge meeting in 1833 and helped to found the Royal Statistical Society in 1834, being Chairman in 1835.

In 1828 he was appointed Lucasian Professor of Mathematics at Cambridge and held this position for eleven years without, it would appear, ever residing in Cambridge or giving a lecture.

He moved to 1 Dorset Street, Portman Square, London, in 1827 and lived there until his death on 18 October 1871.

He was honoured by many foreign learned societies, especially in Italy where his work was greatly esteemed. To quote from the title page of *Passages from the Life of a Philosopher,* he describes himself as

> Charles Babbage, Esq., M.A.,
> F.R.S., F.R.S.E., F.R.A.S., F.Stat.S., Hon.M.R.I.A., M.C.P.S.,
> COMMANDER OF THE ITALIAN ORDER OF ST. MAURICE AND
> ST. LAZARUS, Inst. Imp. (Acad. Moral.) Paris Corr., Acad.
> Amer. Art et Sc. Boston, Reg. Occon. Boruss., Phys. Hist. Nat.
> Genev., Acad. Reg. Monac., Hafn., Mussil., et Divion, Socius,
> Acad. Imp. et Reg. Petrop., Neap., Brux., Patar., Georg. Floren,
> Lyncii Rom., Naut., Phiomath, Paris, Soc. Corr. Etc.

After this brief sketch of the major reference points in Babbage's life, I now turn to the contents of the rest of this book.

Chapter 2 is an attempt to give a background to Babbage's mathematical life by a consideration of the state of the subject in this country at the time of his research. It is now almost banal to say that British mathematics had reached a very low mark by the beginning of the nineteenth century. Hardly any detailed study of this period has been presented, and I have attempted here to give the full picture of what was happening. The result is a study at various levels: in school, university, published books and research papers. More information is obtained by looking at general attitudes to the subject and attempts to reform. An extraordinary situation is revealed. It appears that mathematics was so highly esteemed at Cambridge that it was the only subject for all students at undergraduate level, and yet it was, in

comparison with what was happening on the Continent, virtually neglected for research, industrial or military purposes. Possibly the reason for this paradox is that there is no better way to place people arbitrarily in order of merit than to give them an examination in mathematics, and this was the sole object of the exercise at Cambridge.

Against this background Babbage had to teach himself all the mathematical knowledge he needed. He was very fortunate to have such friends as Herschel, Peacock, Maule, Bromhead and others of a similar mind to his own, but his ability proved to be superior to any of these. By his own discoveries and his general encouragement of mathematical learning, he helped to herald a new era in British mathematics.

The third chapter looks in detail at the formation and work of the Analytical Society, in which Babbage played a considerable part. In particular I discuss the campaign to introduce the differential notation used by the Continental followers of Leibniz to the compatriots of Newton who had generally refused to use it for over a century.

There can be little doubt that Babbage's major mathematical work was in the calculus of functions, and this is studied in some detail in chapter 4. It is shown how, particularly in the two long essays written for the *Philosophical Transactions* of the Royal Society, he took a branch of mathematics barely considered by his predecessors and transformed it into a systematic calculus, the analysis containing some very original stratagems and devices. Justice, it is claimed, has not really been done to his achievement in this field, and I have made an attempt to rectify this.

It was a great pleasure to discover his unpublished book, 'The Philosophy of Analysis', which is examined in chapter 5. There are many interesting items in this work, the most notable being the author's views on algebra, which were far in advance of his time and which anticipated many of the early theories of so-called modern algebra. His analysis of the game of noughts-and-crosses, in which he deduces something of a stochastic process, is also of interest.

Babbage was a versatile mathematician, and chapter 6 surveys some of his other mathematical interests, ranging over geometry, the theory of numbers, probability, and finite and infinite series; but it is not claimed that his mathematical thought was very powerful in these directions.

Chapter 7 is concerned with Babbage's views on notation. Throughout his career he emphasised the vital importance of a good working symbolism. At Cambridge he crusaded successfully for the reformation of notation in the differential calculus. Later he published his views at greater length, not only devising a set of rules that all mathematical notations were to follow but even constructing a kind of notational calculus. We can see continuity from the mathematical to the less mathematical part of his life, when he later devised, as an essential feature of his drawings for the engines, a workable convention which he described as the 'mechanical notation'.

It would not seem proper to discuss Babbage's mathematical work without a consideration of his pioneering work in the computer field. Strictly speaking, the construction of a computer depends on logical rather than specifically mathematical principles, but there is an obvious intimate connection between computers and mathematics. His work on the engines also illustrates the fact that he never abandoned mathematics, but continually used the type of thinking involved in areas that were not so obviously mathematical.

To consider Babbage's work on computers at all, necessarily involves a lengthy discussion, as he strived for practically fifty years in this field. Chapter 8 describes the origin, principle and attempted construction of the difference and analytical engines, without becoming too involved in the mechanisms, while considering all the difficulties that Babbage encountered. His major obstacles were lack of adequate finance, wavering support by the Governments of his day, misunderstanding by the general public, the paucity of precision engineering, and above all his own mind, which so raced ahead of what it was possible to achieve practically that none of these major projects were ever completed.

Finally, I have tried in the last chapter to survey Babbage's major mathematical achievements as seen from the twentieth century, and to assess the influence and importance of his work in the context of the present age.