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Pontryagin Duality and the Structure of Locally Compact Abelian Groups

SIDNEY A. MORRIS

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Preface

Most mathematicians are familiar with the fact that any finitely generated abelian group can be expressed as a direct product of cyclic groups. However, the equally attractive generalization of this to topological groups is known only to a small group of specialists. This is a real pity, as the theory is not only elegant, but also a very pleasant combination of topology and algebra. (It is also related to some problems of diophantine approximation.)

Our aim is to describe the structure of locally compact abelian groups and to acquaint the reader with the Pontryagin-van Kampen duality theorem. This theorem is a deep result and the standard proofs assume a knowledge of measure theory and Banach algebras. In order to make the material accessible to as large an audience as possible I make no such assumption. Indeed even the amount of group theory and topology required is small. Taking the Peter-Weyl theorem as known, I give a simple and, as far as I know, new proof of the duality theorem for compact groups and discrete groups. I then use an approach analogous to that of D. W. Roeder (Category theory applied to Pontryagin duality, *Pacific J.* 52 (1974) 519-527) to extend the duality theorem to all locally compact abelian groups. One of the beauties of this approach is that the structure theory is derived simultaneously.

These notes are based on courses given in 1974 at the University College of North Wales and in 1975 at the University of New South Wales. The former was twenty lectures given to an audience of graduate students and staff while the author was a United Kingdom Science Research Council Senior Visiting

Fellow. The latter was twenty-eight lectures to a final honours class. In 1976 these notes were used as a reading course for final honours students at La Trobe University.

I am indebted to Ronald Brown for persuading me to give my first course on duality theory, for encouraging me to write the material for publication and for his helpful comments. I am extremely grateful to John Loxton and Rodney Nillsen for reading and criticizing the manuscript. Numerous other colleagues, in particular Peter Donovan and Peter Nickolas, made useful comments. I am also grateful to the students in the courses for removal or errors. I must record my thanks to Ian D. Macdonald for his moral support over a number of years and to Edwin Hewitt and Kenneth Ross for their willingness to answer by naive questions. I wish to thank Mesdames Rita Walker, Ulrike Bracken and Olwyn Bradford for their meticulous typing.

These notes are dedicated to Shnookie.

S.A.M.

La Trobe University
Melbourne
1976