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The Topology of Stiefel Manifolds

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Preface

These lectures originated in a course given at Harvard in 1961. Algebraic topology has advanced a long way since that time. Throughout mathematics, the right kind of problem provides the challenge which leads to the improvement of technique and the development of new methods. To a considerable extent, problems about Stiefel manifolds have performed this function in algebraic topology. Thus I felt it might be useful to bring my lectures up-to-date and give some account of what is now known.

The basic theory necessary can be found in a number of text books, such as that of Spanier [132]. At appropriate places I have summarized such additional theory as is needed, with references to the literature, in the hope that these notes may be accessible to non-specialists and particularly to graduate students. Many examples are given and further problems suggested.

The literature on Stiefel manifolds is extensive, as the bibliography at the end of these notes will indicate. The topics I have chosen to discuss in detail are mainly those I have worked on myself, but as well as my own papers I have drawn on those by Adams, Atiyah, Bott and many others. Although much of the material has been published before, in some shape or form, there is a fair amount which has not. The section on further development contains information about work by Friedlander, Gitler, Mahowald, Milgram, Zvengrowski and others which is in process of publication; I am very grateful to those concerned for communicating these results. These notes were read in draft form by Sutherland, Woodward and Zvengrowski, whose comments have been most helpful. I would also like to thank Wilson Sutherland and Emery Thomas for allowing me to quote from joint work, and to thank the American Mathematical Society, Clarendon Press, London Mathematical

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Take the topological product $S^n \times S^n$ of the n-sphere with itself. Remove the diagonal and the antidiagonal. What is left is the space X_n of pairs (x, y) such that $x \neq \pm y$. For what values of n is it possible to make a continuous deformation of X_n into itself in which each such pair (x, y) is deformed into the pair (y, x)? It is known that the deformation is impossible unless $n + 1$ is a power of two; and that the deformation is possible for $n = 1, 3, 7, 15$ and 31; the position for $n = 63, 127, \ldots$ is at present unknown.