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Differentiable Germs and Catastrophes

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Contents

There once lived a man
who learned how to slay dragons
and gave all he possessed
to mastering the art.

After three years
he was fully prepared but,
alas, he found no opportunity
to practise his skills.

Dschuang Dsi.

As a result he began
to teach how to slay dragons.

René Thom.

Foreword

In the summer semester of 1972 I gave a course of lectures on the local theory of differentiable maps at the University of Freiburg. These lectures have formed the basis for the first thirteen chapters of the book, the next three chapters having been written for a summer school organised by the Studienstiftung des deutschen Volkes. My students were responsible for removing many mistakes from the original manuscript which has now been translated into English by L. Lander. He has also made a number of improvements and corrections and provided the last chapter together with its pictures and list of publications. The later chapters discuss a subject which has been the real motivation for writing the book: classical catastrophe theory.

We have both profited greatly from a lecture course on catastrophe theory by K. Jänich, given in Regensburg during the winter semester 1971/72, which contained most of the information and pictures presented in chapter 17.

A small number of copies of the German text of the present book were printed for our students under the title: Der Regensburger Trichter, Band 3, Differenzierbare Abbildungen.

On the pages that follow, the reader will not find any new results or methods. Our purpose is to make it easier for those students, who have properly understood the basic lecture courses on analysis and possess a basic knowledge of algebra, to learn about recent work on differentiable maps, in particular, the mysteries of catastrophe theory.

What are the following pages about?

Let $f : \mathbf{R}^n \to \mathbf{R}^k$ be a differentiable map. What can be said in general about $f^{-1}\{0\}$, that is, about the solution set of a system of non-linear equations? To start with one refers to a theorem of Whitney and Sard's theorem, given in 2.1 and 3.3, in particular one discovers that interesting structure can only be found for 'generic' sets of maps.

Of special interest are the stable differentiable maps, where f is called stable if for a 'small perturbation' $\delta : \mathbf{R}^n \to \mathbf{R}^k$ there are invertible transformations such that the diagram

$$\begin{array}{ccc} \mathbf{R}^n & \xrightarrow{f} & \mathbf{R}^k \\ {\scriptstyle h_1}\downarrow & & \downarrow{\scriptstyle h_2} \\ \mathbf{R}^n & \xrightarrow[f+\delta]{} & \mathbf{R}^k \end{array}$$

commutes. In fact, one expects that natural forms must be described by stable maps because everything in nature is subject to small disturbances. Is 'almost every' map stable? How is the concept of stability to be interpreted?

Any introduction to analysis explains that a differentiable germ $f : (\mathbf{R}, 0) \to (\mathbf{R}, 0)$ with non-vanishing Taylor expansion at the origin can be transformed into the first non-vanishing term by a suitable coordinate change. In higher dimensions, when is a germ determined by a finite part of its Taylor expansion (up to equivalence under coordinate transformations)?

Those are a few of the questions which are discussed below. Perhaps the reader will thereby be encouraged to join in the task of clarifying and understanding some of the ideas of R. Thom.

Regensburg, Spring 1974 Theodor Bröcker