

# Index

Definitions and notation introduced ad hoc for a particular context only are not indexed, but it is hoped that all those which, once introduced, are reused later, have been included. Greek letters are listed separately, after the Latin-alphabet index.

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