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Patrick Du Val
Frontmatter
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Elliptic Functions and Elliptic Curves

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Contents

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Preface

The lectures on which the following notes are based were given in various forms in University College, London, from about 1964 to 1969. Generally they were an optional undergraduate course, containing the substance of Chapters 1-6, and part of Chapter 8. Once or twice they were given to graduate students in geometry, and then included also the bulk of Chapters 9-13. Chapter 7, with the part of Chapter 11 which depends on this, and the cubic transformations in Chapter 8, never figured in the course, but it seemed to me very desirable to add them to the published notes. There is of course much more that I would have liked to include (such as transformations at least of order 5, some study of the connexion between modular relations and the subgroups of finite index in the modular group, a general examination of rectification problems, and the parametrisation of confocal quadrics and of the tetrahedroid and wave surfaces); but a limit of length is laid down for this series of publications, which I fear I have already strained to the utmost.

In my treatment of elliptic functions I have tried above all to present a unified view of the subject as a whole, developing naturally out of the Weierstrass function; and to give the essential rudiments of every aspect of the subject, while unable to enter in very great detail into any one of these. In particular I have been concerned to emphasize the dependence of the properties of the functions on the shape of the lattice; it is for this reason that the modular function is introduced at such an early stage, and that equal prominence is given throughout (except in the context of the Jacobi functions) to the rhombic and the rectangular lattices.

The treatment of the theta functions will be seen to be rather slight. They are in themselves a large subject, of which our study is in a considerable measure independent, since our approach (based on Neville's) to the Jacobi functions obviates any need for the theta functions as a preliminary, except for the expression of invariants such as k, K, J in

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terms of τ or q, i. e. in terms of the lattice shape.

I have kept the analytic apparatus required to a minimum, largely because I am no expert analyst myself; all that I assume ought, I think, to be familiar to any graduate or third-year honours student, and is to be found in any such general textbook as Whittaker and Watson [43] or Copson [5]. For the study of elliptic curves I have of course had to assume some knowledge of algebraic geometry. The general theory sketched in Section 85 can be read up in detail in such works as van der Waerden [38] or Hodge and Pedoe [21]; and the properties of the genus used in Section 89 in any book on algebraic curves, such as Walker [40] or Semple and Kneebone [35]. For any assumed properties of the plane cubic and twisted quartic, probably the best sources are still the two classics of Salmon [32, 33], now available in modern reprints; and for the finite groups $\underline{V}$, $\underline{T}$, $\underline{O}$ etc. perhaps the easiest reference is my own monograph [10].

In conclusion, I would like to express my gratitude to the London Mathematical Society for making this publication possible; to the general editor of the series, Professor G. C. Shephard, for his patience; to Dr D. G. Larman for assistance with the bibliography; and particularly to my wife for her help in reading the proofs.

Istanbul, 1971 Patrick Du Val

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