

Contents

<i>Preface</i>	<i>page</i>	xiii
1 Preliminary results		1
1.1 Rings, modules, and functors		1
1.2 Azumaya–Krull–Schmidt theorem		3
1.3 The structure of rings		4
1.4 The Arnold–Lady theorem		5
2 Class number of an abelian group		9
2.1 Preliminaries		9
2.2 A functorial bijection		14
2.3 Internal cancellation		17
2.4 Power cancellation		19
2.5 Unique decomposition		21
2.6 Algebraic number fields		23
2.7 Exercises		25
2.8 Problems for future research		25
3 Mayer–Vietoris sequences		26
3.1 The sequence of groups		26
3.2 Analytic methods		32
3.3 Exercises		36
3.4 Problems for future research		36
4 Lifting units		37
4.1 Units and sequences		37
4.2 Calculations with primary ideals		40
4.3 Quadratic number fields		44
4.4 The Gaussian integers		45
4.5 Imaginary quadratic number fields		46
4.6 Exercises		48
4.7 Problems for future research		48
5 The conductor		49
5.1 Introduction		49
5.2 Some functors		51
5.3 Conductor of an <i>rtffr</i> ring		53
5.4 Local correspondence		54

viii	<i>Contents</i>	
	5.5 Exercises	60
	5.6 Problems for future research	60
6	Conductors and groups	61
	6.1 <i>Rtffr</i> groups	61
	6.2 Direct sum decompositions	63
	6.3 Locally semi-perfect rings	67
	6.4 Balanced semi-primary groups	70
	6.5 Examples	72
	6.6 Exercises	75
	6.7 Problems for future research	76
7	Invertible fractional ideals	77
	7.1 Introduction	77
	7.2 Functors and bijections	79
	7.3 The square	81
	7.4 Isomorphism classes	90
	7.5 The equivalence class $\{I\}$	91
	7.6 Commutative domains	93
	7.7 Cardinality of the kernels	95
	7.8 Relatively prime to τ	96
	7.9 Power cancellation	98
	7.10 Algebraic number fields	100
	7.11 Exercises	102
	7.12 Problems for future research	102
8	\mathcal{L}-groups	103
	8.1 \mathcal{J} -groups, \mathcal{L} -groups, and \mathcal{S} -groups	103
	8.2 Eichler groups	103
	8.3 Direct sums of \mathcal{L} -groups	106
	8.4 Eichler \mathcal{L} -groups are \mathcal{J} -groups	109
	8.5 Exercises	111
	8.6 Problems for future research	112
9	Modules and homotopy classes	113
	9.1 Right endomorphism modules	113
	9.1.1 The homotopy of G -plexes	113
	9.1.2 Homotopy and homology	118
	9.1.3 Endomorphism modules as G -plexes	120
	9.2 Two commutative triangles	128
	9.2.1 G -Solvable R -modules	129
	9.2.2 A factorization of the tensor functor	131
	9.3 Left endomorphism modules	134
	9.3.1 Duality	137
	9.4 Self-small self-slender modules	142
	9.5 (μ) Implies slender injectives	143
	9.6 Exercises	144
	9.7 Problems for future research	146

		<i>Contents</i>	ix
10	Tensor functor equivalences		148
	10.1 Small projective generators		148
	10.2 Quasi-projective modules		153
	10.3 Flat endomorphism modules		157
	10.3.1 A category equivalence for submodules of free modules		157
	10.3.2 Right ideals in endomorphism rings		161
	10.3.3 A criterion for E -flatness		162
	10.4 Orsatti and Menini's $*$ -modules		163
	10.5 Dualities from injective properties		166
	10.5.1 G -Cosolvable R -modules		167
	10.5.2 A factorization of $\text{Hom}_R(\cdot, G)$		168
	10.5.3 Dualities for the dual functor		169
	10.6 Exercises		171
	10.7 Problems for future research		173
11	Characterizing endomorphisms		175
	11.1 Flat endomorphism modules		175
	11.2 Homological dimension		177
	11.2.1 Definitions and examples		177
	11.2.2 The exact dimension of a G -plex		179
	11.2.3 The projective dimension of a G -plex		180
	11.3 The flat dimension		184
	11.4 Global dimensions		188
	11.5 Small global dimensions		191
	11.5.1 Baer's lemma		191
	11.5.2 Semi-simple rings		194
	11.5.3 Right hereditary rings		195
	11.5.4 Global dimension at most 3		201
	11.6 Injective dimensions and modules		202
	11.6.1 A review of G -cplexes		202
	11.6.2 Injective endomorphism rings		206
	11.6.3 Left homological dimensions		209
	11.7 A glossary of terms		212
	11.8 Exercises		214
	11.9 Problems for future research		218
12	Projective modules		219
	12.1 Projectives		219
	12.2 Finitely generated modules		223
	12.3 Exercises		228
	12.4 Problems for future research		228
13	Finitely generated modules		229
	13.1 Beaumont–Pierce		229
	13.2 Noetherian modules		235
	13.3 Generators		237

x	<i>Contents</i>	
	13.4 Exercises	241
	13.5 Problems for future research	241
14	<i>Rtffr</i> E-projective groups	242
	14.1 Introduction	242
	14.2 The UConn '81 Theorem	245
	14.3 Exercises	248
	14.4 Problems for future research	248
15	Injective endomorphism modules	249
	15.1 G -Monomorphisms	249
	15.2 Injective properties	251
	15.3 G -Cogenerators	258
	15.4 Projective modules revisited	261
	15.5 Examples	262
	15.6 Exercises	263
	15.7 Problems for future research	264
16	A diagram of categories	265
	16.1 The diagram	265
	16.2 Smallness and slenderness	269
	16.3 Coherent objects	272
	16.4 The construction function	272
	16.5 The Greek maps	274
	16.6 Applications	275
	16.6.1 Complete sets of invariants	275
	16.6.2 Unique topological decompositions	276
	16.6.3 Homological dimensions	279
	16.7 Exercises	282
	16.8 Problems for future research	282
17	Diagrams of abelian groups	284
	17.1 The ring $\text{End}_{\mathbb{C}(X)}$	285
	17.2 Topological complexes	286
	17.3 Categories of complexes	288
	17.4 Commutative triangles	291
	17.5 Three diamonds	294
	17.5.1 A diagram for an abelian groups	294
	17.5.2 Self-small and self-slender	296
	17.5.3 Coherent complexes	298
	17.6 Prism diagrams	300
	17.7 Direct sums	301
	17.8 Algebraic number fields	304
	17.9 Exercises	309
	17.10 Problems for future research	311
18	Marginal isomorphisms	312
	18.1 Ore localization	312
	18.1.1 Preliminary concepts and examples	313
	18.1.2 Noncommutative localization	315

<i>Contents</i>		xi
18.2	Marginal isomorphisms	322
18.2.1	Margimorphism and localizations	323
18.2.2	Marginal summands	327
18.2.3	Marginal summands as projectives	329
18.2.4	Projective Q_G -modules	331
18.3	Uniqueness of direct summands	333
18.3.1	Totally indecomposable modules	333
18.3.2	Morphisms of totally indecomposables	334
18.3.3	Semi-simple marginal summands	337
18.3.4	Jónsson's theorem and margimorphisms	339
18.4	Nilpotent sets and margimorphism	342
18.5	Isomorphism from margimorphism	346
18.6	Semi-simple endomorphism rings	353
18.7	Exercises	358
18.8	Problems for future research	360
	<i>Bibliography</i>	362
	<i>Index</i>	368