1 Introduction

The wireless communication industry has been and is still experiencing an exciting era of rapid development. New technologies and designs emerge on a regular basis. The timely adoption of these technologies in real-world systems relies heavily on the accurate prediction of their performance over general wireless fading channels and the associated system complexity. Theoretical performance and complexity analysis become invaluable in this process, because they can help circumvent the time-consuming computer simulation and expensive field test campaigns. These analytical results, usually in the form of elegant closed-form solutions, will also bring important insight into the dependence of the performance as well as complexity measures on system design parameters and, as such, facilitate the determination of the most suitable design choice in the face of practical implementation constraints.

Mathematical and statistical tools play a critical rule in the performance analysis of digital wireless communication systems over fading channels [1]. In fact, the proper utilization of these tools can help either simplify the existing results, which do not allow for efficient numerical evaluation, or render new analytical solutions that were previously deemed infeasible. One popular example is the application of moment generation function (MGF) in the performance analysis of digital communication system over fading channels [2]. With the unified MGF-based analytical framework, the error probability expressions, which usually involve an infinite integration of Gaussian Q-function, are simplified to a single integral of elementary functions with finite limits and, as such, facilitate convenient and accurate numerical evaluation.

Order statistics is an important sub-discipline of statistics theory [3]. Over the years, order statistics has made an increasing number of appearances in the design and analysis of wireless communication systems. Specifically, order statistics have been proven to be valuable tools during the performance analysis of advanced diversity techniques, adaptive transmission techniques, and multiuser scheduling techniques, where the simple but effective engineering principle of "selecting the best" frequently applies. This book aims at providing a coherent and systematic presentation of the applications of order statistics in wireless communication system analysis, which will prepare readers to further explore the potentials of ordered statistics in advanced wireless communication research.

1.1 Order statistics in wireless system analysis

The most common figure of merit governing the performance of a communication system is the signal-to-noise ratio (SNR). (For wireless transmission subject to certain interference, the modified matric signal to interference plus noise ratio (SINR) is often used.) The receiver output SNR, which can be directly related to the instantaneous error rate performance, serves as an excellent indicator of the fidelity of the detection process. As the result of the fading effect associated with multipath wireless channels, the receiver output SNR, usually denoted by γ, will randomly vary over time. In this scenario, the average performance measures should be employed, the evaluation of which would require the statistical distribution of γ. For example, the so-called outage probability performance measure, defined as the probability that the channel quality is too poor for reliable transmission, is calculated as the probability that γ falls below a certain specific threshold $\gamma_{\rm th}$, i.e.

$$P_{\rm out} = \int_0^{\gamma_{\rm th}} p_\gamma(\gamma)\mathrm{d}\gamma, \tag{1.1}$$

where $p_\gamma(\cdot)$ is the probability density function (PDF) of γ. The average error rate of a wireless link can be calculated as

$$\overline{P}_E = \int_0^{\infty} P_E(\gamma)p_\gamma(\gamma)\mathrm{d}\gamma, \tag{1.2}$$

where $P_E(\gamma)$ is the instantaneous error rate of the modulation scheme of interest for a given SNR value γ.

The PDF of γ for the conventional narrowband single-antenna link can be easily derived based on the adopted fading channel models. With the introduction of diversity, adaptation, and scheduling techniques, the statistics of the receiver output SNR become more challenging to obtain. Quite often, such analysis mandates some order statistics results, as the practical best-selection engineering principle frequently applies in the designs. As a classical example, the selection diversity combining technique uses the signal replica with the best quality, i.e. the highest instantaneous SNR, among several available ones for data detection [1]. As such, the receiver output SNR becomes the largest one of several branch SNRs, i.e. $\gamma_{\rm max} = \max\{\gamma_1, \gamma_2, \cdots\}$, the statistics characterization of which is readily available in traditional order statistics literature [4]. Specifically, the PDF of the largest one among L independent and identically distributed (i.i.d.) branch SNRs is given by

$$p_{\gamma_{\rm max}}(x) = L[F_\gamma(x)]^{K-1}p_\gamma(x), \tag{1.3}$$

where $F_\gamma(x)$ and $p_\gamma(x)$ are the common PDF and CDF of γ.

Recently, generalized selection combining (GSC) was proposed as an attractive combining scheme for broadband wireless systems (see for example [5–9]). The basic idea is to select L_c best diversity branches out of a total L available ones, where $L_c < L$, and combine them in the optimal maximum ratio combining (MRC) fashion. The combiner output SNR with GSC becomes the sum of the L_c largest random variables among a total of L ones, the statistics of which were not immediately available over the general fading channel models. Furthermore, the analysis of adaptive combining schemes, where the receiver tries to utilize a minimum amount of combining operations to satisfy a certain threshold requirement, further requires the joint statistics of the partial sums of ordered random variables [10–12]. Such joint statistics also find application in the analysis of multiuser scheduling schemes for OFDMA and MIMO systems. In this book, we will provide a comprehensive treatment of the application of order statistics in the analysis of wireless communication systems and focus in particular on how conventional and new order statistics results help obtain the desired statistics of received output SNR. These statistical results can then be readily utilized to calculate various performance metric of interest by following the standard analytical procedure in [1, 2].

1.2 Diversity, adaptation, and scheduling

Many technologies have been developed to improve the quality and efficiency of wireless communication systems. While providing a coherent presentation of order statistics in wireless communications, we focus on three general classes of wireless techniques, namely diversity, adaptation, and scheduling, all of which are essential building blocks of current and emerging wireless systems and, as such, have received a significant amount of interest from both academia and industry. We adopt a unique analysis-oriented approach in presenting these technologies. Specifically, we present several practical designs for each technology and attack their exact performance and complexity analysis over general fading environment with the help of order statistics results. The primary objective is to accurately quantify the trade-off of performance versus complexity among different design options, which help foster a thorough and in-depth understanding of each technology.

Diversity combining techniques can effectively improve the performance of a wireless communication system operating over a fading environment. Conventional combining schemes have been well-documented in various textbooks on wireless communications [1, 13]. Over the past decade, there have been significant developments in the field of advanced diversity combining techniques for emerging broadband wireless systems. Motivated by the fact that a large number of available paths may exist whereas the system can only afford to process a limited number of paths due to the complexity and cost constraint, the common goal of these newly proposed combining schemes is to efficiently select a subset of strong diversity paths and combine them in the MRC fashion from those that are available [14–16]. The trade-off of performance versus complexity among different schemes mandates the exact analysis on each scheme. This book will provide a comprehensive treatment of several representative advanced diversity-combining schemes.

Channel adaptive transmission and reception techniques can achieve high spectrum and/or power efficiency by effectively exploring the time-varying nature of wireless fading channels. Conventional adaptive transmission techniques vary various transmission parameters with the prevailing channel conditions to achieve highly spectral-efficient transmission while satisfying a certain error rate requirement [13]. With the recent development of adaptive diversity combining techniques, the amount of combining operation will also vary with the fading channel condition. The main objective of the new class of diversity techniques is to adaptively utilize the diversity combiner resource, in terms of active diversity branches and channel estimations, to achieve an overall complexity saving while satisfying a certain output performance requirement [10, 11]. The idea of adaptive combining can be applied to receive diversity, transmit diversity, and diversity in a multicell environment. Furthermore, receiver adaptive combining can also be designed jointly with adaptive transmission to reach a highly efficient transceiver design. This book will provide an analysis-oriented presentation of such designs and their underlying design trade-offs.

Multiuser scheduling can explore the diversity benefit inherent in multiuser wireless systems. The idea is to explore the independent variation of multiuser channels and schedule the users with the best channel conditions to transmit [17,18]. Multiuser scheduling has been shown to be able to acheive great throughput performance. The idea of multiuser scheduling has been incorporated into the emerging cellular system standards. This book will cover some recent developments in multiuser scheduling, including multiuser parallel scheduling [19], and scheduling in multiuser MIMO systems [20, 21]. We focus on those practical designs with low implementation complexity and which, as such, can be directly applied to practical wireless systems. Again, special effort will be made to quantify the various design trade-offs involved, which will benefit both academic research and practicing engineers.

1.3 Outline of the book

The primary focus of this book is the application of ordered statistics in the performance and complexity analysis of various wireless communication technologies. The book also covers three general classes of wireless technologies – namely, advanced diversity techniques, channel adaptive transmission, and reception techniques and multiuser scheduling techniques in the context of emerging MIMO–OFDM systems.

The book is organized as follows. We first summarize the basics of digital wireless communications over fading channels in Chapter 2, which provides the necessary background for the subsequent chapters. Then, the statistical results, more specifically the conventional and new results on the distribution functions of random variables involving order statistics, are presented together with their derivations in Chapter 3. Chapter 4 concentrates on the analysis and design of advanced diversity combining techniques, whereas channel adaptive transmission and reception techniques are presented in Chapter 5. The concept of multiuser scheduling is explored in Chapter 6 and Chapter 7, in the context of OFDMA systems and multiuser MIMO system, respectively.

We strive to achieve an ideal balance between theory and practice. The detailed mathematical derivations for order statistics are grouped into one chapter so that other chapters will focus on the practical design insights. Such an arrangement will allow easy reading and convenient future referencing. Special emphasis will be placed on the important trade-off of performance versus complexity throughout the presentation. Whenever deemed necessary, the associated complexity measures are quantified and plotted together with the performance for clear trade-off illustration.

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2 Digital communications over fading channels

2.1 Introduction

We present a brief summary of digital wireless communications in this chapter. The material will serve as a useful background for the advanced wireless technologies in later chapters. We first review the statistical fading channel models commonly used in wireless system analysis. After that, we discuss digital modulation schemes and their performance analysis over fading channels, including the well-known moment generating function (MGF)-based approach [1]. The basic concept of adaptive modulation and diversity combining will also be presented. While most of the materials of this chapter are reviews of classical results, which can be found in other wireless textbooks, this chapter provides for the first time a thorough treatment of various conventional threshold-based combining schemes, a class of combining scheme enjoying even lower complexity than selection combining (SC). The chapter concludes with a brief discussion of the transmit diversity technique. The discussion of this chapter is by no means comprehensive. The main objective is to introduce some common notation and system models for later chapters. For a thorough treatment of these subjects, the reader may refer to [1, 2].

2.2 Statistical fading channel models

Wireless channels rely on the physical phenomenon of electromagnetic wave propagation, due to the pioneering discoveries of Maxwell and Hertz. Radio waves propagate through several mechanisms, including direct line of sight (LOS), reflection, diffraction, scattering, etc. As such, there usually exist multiple propagation paths between the transmitters and the receivers, as illustrated in Fig. 2.1. In general, when the LOS path exists, as in microwave systems and certain indoor applications, the transmitted radio signal experiences less attenuation. On the other hand, if the LOS path does not exist, the radio signal can still reach the receiver through other mechanisms, but with severe attenuation.

The complicated propagation environment and the unpredictable nature of the propagation process make the modeling of wireless channels very challenging, especially considering the mobility of the transmitter and/or the receiver. To

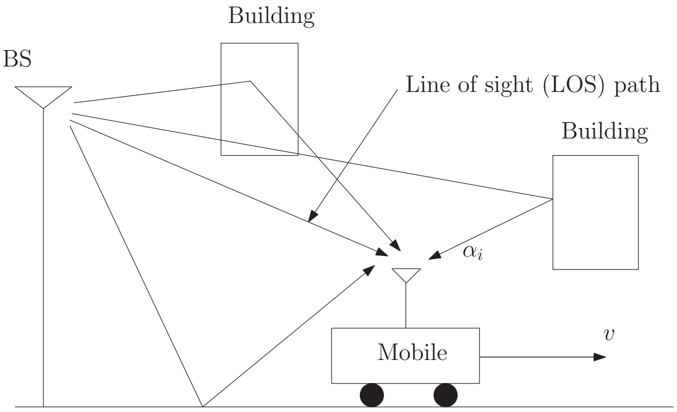

Figure 2.1 Multipath propagation.

avoid the modeling complication associated with the detailed propagation process, the wireless channel is usually characterized by three major effects: (i) path loss, for the general trend of power dissipation as propagation distance increases; (ii) shadowing, for the effects of large objects, such as buildings and trees, along the propagation path; and (iii) fading, resulting from the random superposition of signals from different propagation paths at the receiver. The received signal power variation due to these three effects is demonstrated in Fig. 2.2. In general, path loss and shadowing are called large-scale propagation effects, whereas fading is referred to as the small-scale effect, since the fading effect manifests itself in a much smaller time/spatial scale.

2.2.1 Path loss and shadowing

Path loss characterizes the general trend of power loss as the propagation distance increases. The linear *path loss* is defined as the ratio of the transmitted signal power P_t versus the received signal power P_r, i.e. $PL = P_t/P_r$. Among the different types of path loss models, the log-distance model is the most convenient for high-level system analysis [3]. Specifically, under the log-distance model, path loss at a distance d in dB scale is predicted using the following formula

$$PL(d)\text{dB} = PL(d_0)\text{dB} + 10\beta \log_{10}(d/d_0), \tag{2.1}$$

where d_0 is the reference distance, usually set to 1–10 m for indoor applications and 1 km for outdoor applications, $PL(d_0)$dB is the path loss at d_0 and β is the path loss exponent, which can be estimated by minimizing the mean square error (MSE) between the measurement data and the model. Note that the path loss model only captures the general trend of power dissipation, ignoring the effect of specific surrounding objects or multipaths.

Shadowing characterizes the blockage effect of large objects in the propagation environment. As the size, positioning, and properties of such objects are in

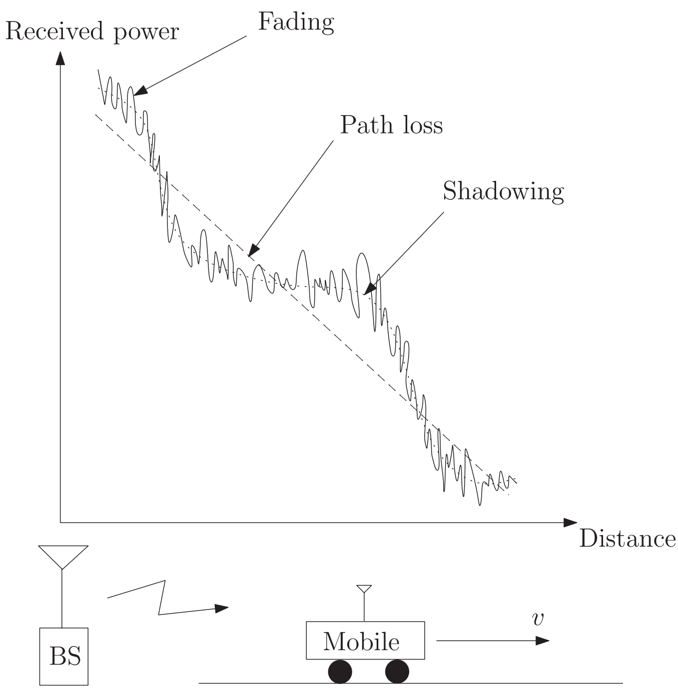

Figure 2.2 Received signal power.

general unknown, the shadowing effect has to be described in the statistical sense. The most popular shadowing model is the log-normal model, which has been emperically confirmed [4]. With the log-normal model, the path loss in dB scale at distance d, denoted by ψ_{dB}, is modeled as a Gaussian random variable with mean value $PL(d)$dB, given by appropriate path loss model, and variance σ_{dB}^2, which varies with the environment. As such, the PDF of ψ_{dB} is given by

$$p_{\psi_d B}(x) = \frac{1}{\sqrt{2\pi}\sigma_{dB}} \exp\left(-\frac{(x - PL(d)dB)^2}{2\sigma_{dB}^2}\right). \tag{2.2}$$

The log-normal model is so named as the path loss in linear scale ψ is a log-normal random variable, with PDF given by

$$p_{\psi}(x) = \frac{10/\ln(10)}{\sqrt{2\pi}x\sigma_{dB}} \exp\left(-\frac{(10\log_{10} x - PL(d)dB)^2}{2\sigma_{dB}^2}\right). \tag{2.3}$$

With the path loss and shadowing model, we can address some interesting system design problems, e.g. for a given transmitting power and target service area, what is the percentage of coverage after considering the path loss and shadowing effects. Assuming a location is covered if the received signal power after shadowing is above the threshold $P_{\min}$, the percentage of coverage can be calculated by averaging the probability that the received signal power at a

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distance r is less than $P_{\min}$ over the target area. Mathematically, we have

$$C = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R \Pr[P_r(r) > P_{\min}] r \mathrm{d}r \mathrm{d}\theta, \tag{2.4}$$

where R is the radius of the cell. Noting that the coverage probability under the combined log-distance path loss model and log-normal shadowing model is given by

$$\Pr[P_r(r) > P_{\min}] = \int_{-\infty}^{P_t - P_{\min}} p_{\psi_d B}(x) \mathrm{d}x, \tag{2.5}$$

the coverage percentage C can be shown to be given by

$$C = Q(a) + \exp\left(\frac{2 - 2ab}{b^2}\right) Q\left(\frac{2 - ab}{b}\right) \tag{2.6}$$

where

$$a = \frac{P_{\min} - P_t + PL(d_0)\mathrm{dB} + 10\gamma \log_{10}(R/d_0)}{\sigma_{dB}}, b = \frac{10\gamma \log_{10}(e)}{\sigma_{dB}},$$

and $Q(\cdot)$ is the Gaussian Q-function[1], defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} \mathrm{d}t. \tag{2.7}$$

2.2.2 Multipath fading

Fading characterizes the effect of random superposition of signal copies arriving at the receiver from different propagation paths. These signal replicas may add together constructively or cancel one another, which leads to a large variation in received signal strength. To better demonstrate the process, let us assume the following bandpass signal is transmitted over a wireless channel

$$s(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\}, \tag{2.8}$$

where $u(t)$ is complex baseband envelope. Due to multipath propagation, the received signal becomes

$$r(t) = \mathrm{Re}\left\{\sum_{n=0}^{N(t)} \alpha_n(t) u(t - \tau_n(t)) e^{j2\pi f_c (t - \tau_n(t)) + \phi_{D_n}(t)}\right\}. \tag{2.9}$$

where $N(t)$ is the number of paths, $\tau_n(t)$ is the delay, $\alpha_n(t)$ is the amplitude, and $\phi_{D_n}(t)$ is the phase shift, all for the nth path at time t. Note that the phase shift $\phi_{D_n}(t)$ is related to the Doppler frequency shift as $\phi_{D_n}(t) = \int_t 2\pi f_{D_n}(t)\mathrm{d}t$. After some manipulations while focusing on the complex baseband input and output

[1] The Gaussian Q-function is related to the complementary error function $\mathrm{erfc}(\cdot)$ by $\mathrm{erfc}(x) = 2Q(\sqrt{2}x)$ and $Q(x) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$.