

A Guide to Feedback Theory

Do you need to understand feedback? Perhaps you're a little rusty on theory basics? Dig in to this self-contained guide for an accessible and concise explanation of the fundamentals.

- Distills the relevant essence of linear system theory, calculus, differential equations, linear algebra, basic physics, numerical methods, and complex analysis and links them back to an explanation of feedback theory.
- Provides a tight synthesis of analytical and conceptual understanding.
- Maintains a focus on common use cases.

Whether you are a struggling undergraduate, a doctoral student preparing for your qualifying exams, or an industry practitioner, this easy-to-understand book invites you to relax, enjoy the material, and follow your curiosity.

JOEL L. DAWSON is an entrepreneur and former MIT professor. He is a 2009 recipient of the PECASE Award, the highest honor bestowed by the US government on young scientists and engineers. His last start-up company, Eta Devices, Inc., was a Technology Pioneer of the 2015 World Economic Forum and acquired by Nokia in 2016. His current start-up company is TalkingHeads Wireless, Inc.

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“Feedback theory is an intrinsically mathematical discipline in which one can feel either submerged by formulae or driven to use blind computer simulations that hide insight. Dawson’s approach is to extract visceral meaning out of this tangle, arguing that a deep understanding of dynamic stability criteria can free the designer from “equational overload” and lead to incisive selection of the right mathematical tool for the job at hand.”

Stephen D. Senturia, *Massachusetts Institute of Technology*

“Feedback is perhaps the most foundational concept for electronics and control systems in general, but it is often covered for specific circuits for the former, and in terms of theoretical concepts for the latter. This book provides us with a unique perspective on how feedback theory in general relates to practical systems and electronics applications.”

Larry Pileggi, *Carnegie Mellon University*

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For Elías

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Preface

This is a book about classical feedback control and, along the way, a review of the foundational linear system theory that is at the core of the electrical engineering discipline. But there are many, many books on these subjects. What makes this book different?

The answer is that this book, in its structure, content, and style, targets four very specific groups of people. In no particular order:

- **The drowning undergraduate.** In this, I take inspiration from *Signals and Systems Made Ridiculously Simple* by Zoher Z. Karu (ZiZi Press, 1995). While very different in style, his book was a godsend to us undergraduates at MIT for its simple, short, clear explanations. More deeply, the book unobtrusively guided students *toward* what is important and *away* from what is not. With years to reflect, I look back on that unpretentious volume with affection, gratitude, and respect.
- **The superstar undergraduate.** Perhaps you are absolutely crushing your exams and reaping praise from your professors accordingly. This is a good feeling, and sometimes a just reward for hard work. But a peculiarity of our species can manifest quite strongly here: dominating exam performance is some mix of (1) deep understanding of the material, on one hand, and (2) on the other hand, a virtuosity in pure test taking that owes much to a kind of supercharged social intelligence. What is really astonishing, especially in a competitive, “elite” college environment, is just how far the latter can take you with little development of the former. Given how we structure higher education, an overreliance on (2) in some students is inevitable and natural and to be neither condemned nor praised. But if you have taken up this book, take the opportunity in reading it to do a little self-assessment and reflection. It is the deep understanding of (1) that you need to build useful and interesting machines.

- **The graduate student preparing for their qualifying exams.** Many doctoral programs kick things off in the first or second year with a weighty set of exams designed to probe a candidate's understanding of basic undergraduate material. The weeks or months preparing for a doctoral exam can be extremely rewarding: one discovers that the “basic” undergraduate material is both broad and deep. Moreover, this is a time when many emerging scholars and practitioners come into their own as true self-learners. When I had the first discussions with Julie Lancashire at Cambridge University Press about this book, my original inspiration was to write the book I wished had existed when I was studying for my own qualifying exams. I have tried to stay true to that spirit throughout.
- **A subset of industry practitioners.** Technology in mature and high-volume industries doesn't change very often. In cellular wireless networks, for example, at the time of this writing, the dominant power amplifier architecture continued to be the Doherty structure, invented at Bell Labs in 1936. It is very possible to be a professional in mature fields *and build a successful career*, while having only a loose grasp of the underlying physics. Trade-offs in the performance space can be memorized (“You know, Bill, if you make it faster, it's definitely going to be less stable. There are always trade-offs, my friend, always trade-offs”), and vocabulary can be used accurately by people who have a superficial understanding of where it all comes from. The danger here is that technological paradigm shifts do happen from time to time. If you understand the foundations of your field, you will be able to acquire a similar understanding of the new paradigm through diligent self-study. If your understanding has always been superficial, however, you are helpless and exposed in the face of major change. This can be quite scary.¹ If you find yourself vulnerable to this type of situation, the goal of this book is to be a friendly example of a new type of understanding. Relax and have fun with the material, and allow yourself to follow your curiosity.

My hope for the reader is that you find this book as enjoyable and informative to read as it was to write.

¹ If you are very lucky, old enough, and happen to look the part, you might be able to pull a convincing turn as a curmudgeon of the Old School who simply prefers things that have Stood the Test of Time.

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I am immensely grateful to Prof. James K. Roberge (1938–2014). At various times he was a mentor, teacher, friend, and finally colleague to me on the faculty at the Massachusetts Institute of Technology. I first met him at a time in my life when my curiosity about feedback and electrical engineering far exceeded my knowledge. It may be that there was no better person in the world to meet in that circumstance than him.

Finally, I give heartfelt thanks to my family and friends. They have been a source of encouragement, inspiration, and support through many ups and downs over the years. This book would not have been possible without them.