

Index

- *-reverse polynomials, 201
- Abdi's q -Bessel polynomials, 144
- addition theorem, 37
- addition theorem for Legendre polynomials, 73
- addition theorem from continued fractions, 29
- Airy function, 6
- Al-Salam–Carlitz polynomials, 148
 - continued fraction, 150
 - explicit form, 148, 150
 - generating function, 148, 150
 - lowering and raising operators, 150
 - orthogonality relation, 148, 150
 - q -Sturm–Liouville equation, 150
 - recurrence relation, 148, 150
- Al-Salam–Chihara polynomials, 178, 294
 - explicit form, 178
 - generating function, 179–181
 - moment representations, 180
 - orthogonality relation, 178
 - raising and lowering operators, 179
 - recurrence relation, 178
- Al-Salam–Ismail polynomials, 170
- Al-Salam–Verma rational functions, 156
- Aleksandrov measures, 203
- annihilation operator, 41
- arithmetic properties of the zeros, 80
- Askey–Wilson divided difference operator, 11
- Askey–Wilson equation, 185
 - Askey–Wilson q -Sturm–Liouville, 185, 186
- Askey–Wilson operator, 11
- Askey–Wilson polynomials, 181
 - connection relation, 183
 - explicit form, 182
 - generating function, 182
 - orthogonality, 182
 - raising and lowering operator, 183
 - recurrence relation, 182
- associated Askey–Wilson polynomials, 195
- associated polynomials, 22
- associated q -ultraspherical polynomials, 172
- averaging operator, 11
- balanced, 7, 10
- basic hypergeometric series, 10
- Bessel differential equation, 4
- big q -Jacobi polynomials
 - explicit form, 151
 - generating function, 152
 - lowering operator, 152
 - orthogonality relation, 152
 - raising operator, 153
 - recurrence relation, 152
 - Rodrigues formula, 153
 - Sturm–Liouville equation, 153
- bilateral basic hypergeometric function, 14
- bootstrap, 15
- Carathéodory function, 204
- Carleman criteria, 271
- Carleman criterion for Stieltjes moments, 281
- Casorati determinant, 22, 48
- chain sequence, 245
- Challifour problem, 278
- Charlier polynomials, 131
- Chebyshev polynomials, 67
- Chen–Ismail integrals, 288
- Chen–Ismail polynomials, 287
- Christoffel kernels, 205
- Christoffel numbers, 23
- Christoffel's theorem, 30
- Christoffel–Darboux formula, 19, 62, 205
- Chu–Vandermonde sum, 7
- class \mathcal{S} of Stieltjes transforms, 284
- comparative asymptotics, 214
- complete elliptic integrals, 7
- completely monotonic, 270
- confluent hypergeometric function, 4
- connection coefficients, 34
- connection relations, 78
- continued J -fraction, 27

- continuous dual q^{-1} -Hahn polynomials, 293
 continuous dual q -Hahn polynomials, 293
 continuous q -Jacobi polynomials, 187
 - discriminant, 190
 - explicit form, 187
 - generating function, 188
 - linearization of products, 190
 - orthogonality, 187
 - raising and lowering operators, 188
 - recurrence relation, 187
 - Rodrigues-type formula, 188
 convergents of a continued fraction, 27
 convolution structure, 38
 convolution-type representation for Jacobi polynomials, 52
 creation operator, 41
 de Boor–Saff duality, 49
 degenerate, 269
 determinate, 270
 difference equation for general discrete polynomials, 139
 difference equations, 138
 differential equation for Legendre polynomials, 72
 differential equations for orthogonal polynomials on the unit circle, 224
 differential recurrence relation, 75
 Dirichlet–Mehler representation for ultraspherical polynomials, 66
 discrete discriminant, 45, 140
 discrete Schrödinger operator, 26
 discriminant, 44
 discriminant relative to T , 45
 dual convolution structure, 38
 dual Hahn polynomials, 133
 - difference equation, 134
 - explicit form, 133
 - generating functions, 134
 - orthogonality relation, 133
 - recurrence relation, 133
 - Rodrigues-type formula, 134
 dual system, 48
 electrostatic equilibrium in unit disk, 226
 electrostatic equilibrium theorem, 46
 entropy integral, 278
 Erdős class \mathcal{E} , 213
 Euler integral representation, 3
 exceptional case of Jacobi polynomials, 55
 explicit formula for ultraspherical polynomials, 65
 extreme point, 276
 Freud weights, 272
 Friedrichs measure, 282
 functions of the second kind, 47, 73
 Gauss sum, 7
 Gegenbauer addition theorem, 67
 generalized translation operator, 38
 - generating function for Chebyshev polynomials, 68
 - generating function of Legendre polynomials, 71
 - generating functions, 134
 - generating functions for Jacobi polynomials, 56
 - generating functions for ultraspherical polynomials, 66
 - growth
 - exponential, 285
 - polynomial, 285
 - Hahn polynomials, 131
 - difference equation, 133
 - explicit representation, 131
 - orthogonality relation, 132
 - Hamburger moment problem, 269
 Hankel determinant, 16
 Hankel matrix, 18, 21
 Hausdorff moment problem, 270
 Heine integral representation, 17
 Hellmann–Feynman theorem, 244
 Herglotz functions, 274
 Hermite functions, 84
 Hermite polynomials
 - differential equation, 76
 - differential recurrence relation, 76
 - explicit formula, 74
 - generating functions, 77
 - orthogonality, 74
 - Hille–Hardy formula, 86
 hypergeometric differential equation, 3
 indeterminate, 270
 index of determinacy, 277
 integral
 - Ismail–Valent, 289
 - integral relations, 79
 integral representations of Jacobi polynomials, 88
 Ismail’s q -Bessel polynomials, 145
 Ismail’s q -Lommel polynomials, 143
 Ismail–Mulla polynomials, 176
 Ismail–Stanton integral, 184
 Ismail–Valent integral evaluations, 289
 Ismail–Valent polynomials, 288
 IVY polynomials, 289
 Jacobi matrix, 19, 26
 Jacobi polynomials
 - connection relation, 54
 - definition, 51
 - differential equation, 53
 - discriminant, 54
 - lowering operator, 52, 54
 - orthogonality, 51
 - raising operator, 52
 - Rodrigues formula, 53
 - three-term recurrence relation, 53
 - Jacobi triple product identity, 14
 kernel polynomial, 21
 Kibble–Slepian formula, 85

- Koornwinder polynomials, 33
- Krawtchouk polynomials, 135
- difference equation, 135
 - orthogonality relation, 135
 - raising and lowering operators, 136
 - recurrence relation, 135
 - Rodrigues-type formula, 136
- Krein measure, 282
- Krein parametrization, 284
- Krein's theorem, 282
- ladder operators for Legendre polynomials, 72
- Laguerre method, 248
- Laguerre polynomials
- differential equation, 75
 - explicit formula, 74
 - generating function, 77
 - orthogonality, 74
 - Rodrigues formula, 76
- Laplace first integral for ultraspherical polynomials, 65
- Legendre polynomials, 71
- linearization coefficients, 34
- little q -Jacobi polynomials, 152
- discriminant, 154
 - explicit form, 152
 - orthogonality relation, 152
- Lloyd polynomials, 136
- logarithmic potential, 46
- Lommel polynomials, 141
- continued fraction, 141
 - explicit form, 141
 - orthogonality relation, 142
 - recurrence relation, 141
- lowering operator, 41
- lowering relation, 39
- Markov's theorem, 27, 243, 272
- Markov's theorem, generalized, 243
- matrix polynomial, 307
- matrix three-term recurrence formula, 309
- Mehler formula, 86
- Meixner polynomials, 129
- orthogonality relation, 129
 - recurrence relation, 129
- minimal solution, 27
- modification of orthogonality measures by a discrete part, 32
- moment problem, 269
- determinate, 270
 - Hamburger, 269
 - Hausdorff, 270
 - indeterminate, 270
 - Stieltjes, 269
- moments, 16
- monic orthogonal polynomials, 16
- multiple q -shifted factorials, 10
- multiplication formulas, 78
- multiplication operator, 26
 - multishifted factorial, 3
 - n -canonical, 276
 - N -extremal, 275
 - Naimark's theorem, 276
 - Nevai class N , 213
 - Nevanlinna functions, 274
 - Nevanlinna matrix, 274
 - Nevanlinna matrix for Stieltjes moment problem, 282
 - Nevanlinna parameters for Al-Salam–Chihara polynomials, 294
 - Nevanlinna parametrization, 275
 - Nevanlinna polynomials, 273
 - Nevanlinna theorem, 273
 - Nicholson-type formula, 85
 - nonterminating, 10
 - numerator polynomials, 22
- operational representation, 79
- orthonormal polynomials, 17
- Parseval's formula, 270
- Perron–Stieltjes inversion formula, 1, 278
- Pfaff–Kummer transformation, 8
- Pfaff–Saalschütz theorem, 7
- Pick function, 274
- Pincherle's theorem, 28
- plane wave expansion, 55
- Poisson kernel, 20
- Poisson kernel for Hermite polynomials, 86
- Poisson kernel for Laguerre polynomials, 86
- polynomial killer, 25
- polynomial modification of orthogonality measures, 30
- polynomials
- Al-Salam–Chihara for $q > 1$, 294
 - Chen–Ismail, 287
 - continuous dual q^{-1} , 293
 - discrete q -Hermite, 303
 - Ismail–Valent–Yoon (IVY), 289
 - q^{-1} -Hermite, 297
 - q^{-1} -Meixner–Pollaczek, 295
 - q -Laguerre, 300
 - q -Meixner, 299
 - Stieltjes–Wigert, 301
 - symmetric Al-Salam–Chihara, 296
 - Valent–Berg, 285
- positivity results, 81
- power sums of zeros, 20
- product formula, 37
- product rule for D_q , 8
- q -analogue of integration by parts, 9
- q -analogues of the exponential function, 12
- q -Bessel functions, 13
- q -difference equation for general q -polynomials, 147
- q -discriminant, 45
- q -gamma function, 12

- q -Hahn polynomials, 154
 - explicit form, 154
 - generating function, 155
 - orthogonality relation, 154
 - raising and lowering operator, 155
 - recurrence relation, 154
 - Rodrigues formula, 155
 - Sturm–Liouville equation, 155
- q -Hermite polynomials, 157
 - explicit form, 158
 - generating functions, 158
 - linearization of products, 158
 - orthogonality relation, 158
 - product formulas, 194
 - q -integral representation, 160
- q -integral, 8
- q -Jacobi polynomials, 151
- q -Pollaczek polynomials, 175
- q -Racah polynomials, 190
 - difference equation, 191
 - generating function, 192
 - orthogonality relation, 191
 - recurrence relation, 191
- q -shifted factorials, 10
- q -Sturm–Liouville problems, 146
- q -ultraspherical polynomials, 161
 - addition theorem, 184
 - bilinear generating functions, 167
 - connection relation, 165
 - explicit form, 162
 - generating function, 162
 - linearization of products, 166
 - lowering operator, 164
 - orthogonality relation, 162
 - raising operator, 164
 - recurrence relation, 161
- quadrature formula, 23
- raising and lowering operators, 132, 134
- raising and lowering operators for q -polynomials, 147
- raising and lowering operators for general discrete polynomials, 138
- raising operator, 41
- raising relation, 39
- Rakhmanov class \mathcal{R} , 213
- Ramanujan ${}_1\psi_1$ sum, 14
- Ramanujan continued fraction, 171
- ratio asymptotics, 215
- rational modification of orthogonality measures, 30
- recurrence relation for Chebyshev polynomials, 69
- recurrence relation for ultraspherical polynomials, 65
- reducibility to scalar case, 309
- resultant, 44
- reverse polynomial, 201
- Riesz's theorem, 276
- right inverse of the Askey–Wilson operator, 69
- Rodrigues formula, 42, 76
- Rodrigues formula for Legendre polynomials, 72
- Rodrigues formula for ultraspherical polynomials, 65
- Rogers–Ramanujan identities, 168
- Schur polynomials, 169
- semiclassical, 49
- separation theorem, 23
- shifted factorial, 2
- Sonine's first integral, 5
- Sonine's second integral, 5
- spectral theorem for orthogonal polynomials, 24
- Stieltjes moment problems, 269, 280
- Stirling formula, 8
- Sturm comparison theorem, 247
- support of a matrix of measures, 307
- symmetric Hamburger moment sequence, 280
- Szegő class, 207
- Szegő condition, 207
- Szegő estimate, 213
- Szegő function, 207
- Szegő mapping theorem, 201
- Szegő recurrences, 202
- terminating, 3, 10
- theta functions, 15
- three-term recurrence relation, 18
- three-term recurrence relation for Legendre polynomials, 71
- Toda lattice equations, 31
- Toeplitz determinants, 199
- transfer matrix, 202
- true interval of orthogonality, 25
- Turán measures, 218
- ultraspherical differential equation, 65
- ultraspherical plane wave expansion, 66
- universal measure, 219
- Uvarov's theorem, 30
- Valent's conjecture, 293
- Valent–Berg polynomials, 285
- Verblunsky coefficients, 202
- Verblunsky's theorem, 203
- Verma's expansion
 - first, 35
 - second, 35
- weight matrix, 308
- Wendroff's theorem, 33
- Widom's zero theorem, 218
- Wilson operator, 3
- Wimp polynomials, 143
- zeros
 - associate Laguerre polynomials, 267
 - Hermite polynomials and functions, 264
 - Jacobi polynomials, 249
 - Laguerre polynomials, 260
 - ultraspherical polynomials, 255