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A Brief History of Unification

> "Pure logical thinking cannot yield us any knowledge of the empirical world; all knowledge of reality starts from experience and ends in it."
>
> —Albert Einstein*

Unification is a common theme that runs through the development of particle physics. Progress is made when one discovers an underlying unifying principle that connects different particles, different forces, or different phenomena. One of the early examples is the concept of iso-spin. Heisenberg [2] introduced it to explain the very similar nuclear properties of the proton and of the neutron despite the fact that one is charged and the other is neutral. Thus, here one postulates an $SU(2)$ internal symmetry group with generators T_a $(a = 1,\ 2,\ 3)$ which satisfy the algebra

$$[T_a,\ T_b] = i\epsilon_{abc}T_c, \tag{1.1}$$

where ϵ_{abc} is $+1$ when a, b, and c are cyclic, -1 when they are acyclic, and vanishes otherwise, and the proton (p) and the neutron (n) belong to the doublet representation of this group so that

$$N \equiv \begin{pmatrix} p \\ n \end{pmatrix} \to \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}, \tag{1.2}$$

where the column after the arrow gives T_3 quantum numbers of the components. The column vector with p and n states is often called the nucleon, to emphasize that the proton and the neutron are coming from a common multiplet. Similarly, the three pseudo-scalar mesons $\pi^{\pm}$ and π^0 can be thought of as components of a $T = 1$ multiplet ϕ_a where

$$\pi^{\pm} = (\phi_1 \mp i\phi_2)/\sqrt{2},\ \pi^0 = \phi_3. \tag{1.3}$$

* From the Herbert Spencer Lecture 1933 as cited by A. Salam [1].

The pion–nucleon system then can be described by the interaction Lagrangian

$$\mathcal{L}_{\pi\pi N} = g_{\pi\pi N}\bar{N}\gamma_5\frac{\tau_a}{2}N\phi_a, \tag{1.4}$$

where τ_a $(a = 1,\ 2,\ 3)$ are the Pauli matrices. It is easily checked that this interaction is invariant under global iso-spin transformations. The interactions $\bar{p}p\pi^0$, $\bar{n}n\pi^0$, and $\bar{p}n\pi^+$ are now governed by a single coupling constant.

A more potent illustration is the classification of the pseudo-scalar mesons and baryons. Thus, the $J^{\mathrm{P}} = 0^-$ mesons consisting of π^+, π^0, π^-, K^+, K^0, $\overline{K^0}$, K^-, and η^0 can be grouped into the octet representation [3–5] of an $SU(3)$ flavor group with assignment of iso-spin T and a new quantum number called hypercharge Y as follows:

$$\begin{array}{rcc} 0^- \text{ mesons} & T & Y \\ \pi^+, \pi^0,\ \pi^- & 1 & 0 \\ K^+,\ K^0 & \frac{1}{2} & \frac{1}{2} \\ \overline{K^0},\ K^- & \frac{1}{2} & -\frac{1}{2} \\ \eta^0 & 0 & 0 \end{array} \tag{1.5}$$

where the hypercharge is defined so that

$$Y = \frac{1}{2}(S + N_B). \tag{1.6}$$

Here, S is the strangeness quantum number and N_B is the baryon number, and the electric charge of a particle is given by

$$Q = T_3 + Y\,. \tag{1.7}$$

Similarly, the spin 1/2 baryons p, n, Σ^+, Σ^0, Σ^-, Ξ^0, and Ξ^- can also be classified in the octet representation of $SU(3)$ with the iso-spin and hypercharge assignments similar to the case for the pseudo-scalar bosons as shown below:

$$\begin{array}{rcc} \text{spin } \frac{1}{2} \text{ baryons} & T & Y \\ \Sigma^+,\ \Sigma^0,\ \Sigma^- & 1 & 0 \\ p,\ n & \frac{1}{2} & \frac{1}{2} \\ \Xi^0,\ \Xi^- & \frac{1}{2} & -\frac{1}{2} \\ \Lambda^0 & 0 & 0 \end{array} \tag{1.8}$$

Further, the interactions of the octet of baryons with the octet of pseudo-scalar mesons can be described by just two (F-type and D-type) coupling constants. This means that the couplings of interactions such as $\bar{N}\vec{\tau}N\cdot\vec{\pi}$ and $\bar{\Lambda}\Lambda\eta^0$, $\bar{N}\Lambda K$

become related, which is an enormous simplification over an otherwise huge number of possible couplings in the absence of a symmetry principle. Of course, the $SU(3)$ symmetry like the iso-spin symmetry is not exact, but simple assumptions on how the symmetry breaks allow one to derive sum rules known as Gell–Mann–Okubo mass relations [6] among the masses within the multiplets, and these sum rules are in reasonably good agreement with experimental data on the pseudo-scalar meson masses and on the baryon masses.

The $SU(3)$ symmetry also helps in the classification of the meson–baryon resonances. Since the meson–baryon resonances must decay into mesons and baryons which belong to octet representations, one expects that the meson–baryon resonances are likely to belong to one of the irreducible representations in the product of meson and baryon octets. Thus,

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10^* \oplus 27. \tag{1.9}$$

Experimentally, the meson–baryon resonance $N^*(1232)$ (where the number in the parentheses indicates the mass of the resonance in MeV) is a $J^P = 3/2^+$ and $T = 3/2$ object. The iso-spin assignment implies that the resonance should belong to either the 10-plet or the 27-plet, with the 10-plet being the simplest possibility. Indeed, observation of other $J^P = 3/2^+$ resonances allows one to fill in the full 10-plet as shown below. Historically, the existence of Ω^- as a missing piece in the 10-plet was a prediction which was subsequently verified.

$$\begin{array}{llll} J^P = \dfrac{3}{2}^+ & \text{Resonances} & T & Y \\ & N^* & \dfrac{3}{2} & \dfrac{1}{2} \\ & Y^* & 1 & 0 \\ & \Xi^* & \dfrac{1}{2} & -\dfrac{1}{2} \\ & \Omega^- & 0 & -1 \end{array} \tag{1.10}$$

In the above, the classification of multiplets as belonging to irreducible representations of $SU(3)$ allowed us to make further predictions regarding the nature of meson–baryon resonances. However, one may ask if the octets of mesons and baryons are truly fundamental. In 1964, Gell-Mann and Zweig [7–9] proposed that the mesons and baryons could themselves be composed of something more fundamental such as particles which belong to the 3-plet representation of $SU(3)$ (quarks) and their conjugates 3^* (anti-quarks) with iso-spin and hypercharge assignments $(T)^Y$ as follows:

$$3 = \left(\frac{1}{2}\right)^{1/6} + (0)^{-1/3} \tag{1.11}$$

$$3^* = \left(\frac{1}{2}\right)^{-1/6} + (0)^{1/3}. \tag{1.12}$$

Using the above, one can generate pseudo-scalar meson states so that

$$3 \otimes 3^* = 1 \oplus 8. \tag{1.13}$$

We can label the quarks u, d, and s with the quantum numbers as follows:

Quark	Q	T	T_3	Y
u	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$
d	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$
s	$-\frac{1}{3}$	0	0	$-\frac{1}{3}$

(1.14)

It is then easily seen that the iso-spin and hypercharge assignments for the pseudo-scalar meson octet follow directly from the iso-spin and hypercharge assignments of the 3 and 3^*. Similarly, the baryons can be constructed from the product of three 3-plets so that

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10. \tag{1.15}$$

Again, the iso-spin and hypercharge assignment of the octet of baryons follows from the iso-spin and hypercharge assignment of the quarks.

The above picture, however, causes a problem. In this picture one has mesons as bound states of quarks and anti-quarks, i.e., $q\bar{q}$ states while the baryons are bound states of three quarks, i.e., qqq states. However, qq states and $qqqq$ states can also form. Thus, qq bound states would appear as fractionally charged particles but there is no experimental evidence for them. Further, the N^* which has $J = 3/2$ and $T = 3/2$, contains a doubly charged state N^{*++}, which can be thought of as made up of three u quarks, i.e., the state $u^\uparrow u^\uparrow u^\uparrow$, i.e., with all spins up. This makes the spin wave function totally symmetric. When coupled with the fact that N^{*++} is the ground state of three quarks, the wave function of the three quarks becomes totally symmetric – in contradiction with the Fermi–Dirac statistics. The solution to the problem is offered by the introduction of a hidden quantum number [10,11], generally known as color. Specifically, we can introduce a color group $SU(3)_C$ whose fundamental representation contains three states, i.e., states which we can label as red, green, and blue or simply 1, 2, and 3. In this case we can write $N^{*++} \sim \epsilon_{abc} u_a(x_1) u_b(x_2) u_c(x_3)$, where we have suppressed the spins. This makes the state consistent with Fermi–Dirac statistics.

Even more central to the development of particle physics are local symmetries, which appear to be far more fundamental. Local symmetry includes the principle of gauge invariance, and this principle has played a key role in the development of theories of particle physics. Examples of these are the Maxwell–Dirac theory, the Yang–Mills–Dirac theory, and the Einstein theory of gravitation However, a major weakness of gauge theories is that while the principle of gauge invariance determines the self-interactions of the gauge fields and also constraints the interactions of the gauge fields with matter fields, the gauge principle is not strong enough to determine the nature of matter fields themselves. Thus, consider the Dirac–Maxwell system described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \sum_i \bar{\psi}_i \left[\frac{1}{i}\gamma^\mu(\partial_\mu - igQ_iA_\mu) + m_i\right]\psi_i. \tag{1.16}$$

Here, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength and ψ_i are spin 1/2 Dirac fields which carry the $U(1)$ gauge charges Q_i. The equation of motion for the gauge field in this case reads

$$\partial_\nu F^{\mu\nu} = gJ^\mu$$
$$J^\mu = \sum_i Q_i\bar{\psi}_i\gamma^\mu\psi_i, \tag{1.17}$$

The principle of gauge invariance determines the interaction of the gauge field A_μ with the spin half fields ψ_i but is not powerful enough to determine the nature and the number of fields or the charges Q_i. This turns out to be a major weakness in our understanding, i.e., we lack a principle that can determine the type and the number of fields that enter into Eq. (1.16). This problem is persistent and permeates essentially all sectors of particle theory. We consider two further examples: Yang–Mills theory [12–14] and the Einstein theory. For the coupled Yang–Mills–Dirac system for a gauge group G with generators T_a which satisfy the algebraic relation $[T_a, T_b] = iC_{abc}T_c$ where C_{abc} are the structure constants of the gauge group, and A^a_μ are the gauge fields belonging to the adjoint representation of the gauge group, Eq. (1.17) is replaced by

$$\partial_\nu F^{\mu\nu}_a + gC_{abc}A_{\nu b}F^{\mu\nu}_c = gJ^\mu_a. \tag{1.18}$$

Here $F^{\mu\nu}_a$ are the field strengths given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gC_{abc}A^b_\mu A^c_\nu, \tag{1.19}$$

while the source J^μ_a contains the matter fields, and is given by

$$J^\mu_a = \sum_i Q_i\bar{\psi}_i\gamma^\mu T_a\psi_i. \tag{1.20}$$

where ψ_i are assumed to be spin 1/2 fields which belong to the fundamental representation of the gauge group. Using Eq. (1.19) in Eq. (1.18), we find that the Yang–Mills gauge invariance fully determines the self-interactions of the gauge fields A^a_μ. However, the Yang–Mills gauge invariance does not fully determine the right-hand side of Eq. (1.18), which is given by Eq. (1.20) and depends on a number of fields ψ_i which can be chosen in an arbitrary fashion.

The same problem reappears when we consider gravity described by the Einstein theory, which is another example of a gauge theory. Here, the analogue of Eq. (1.18) is the Einstein field equations, which read

$$R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = -\kappa^2 T^{\alpha\beta}. \tag{1.21}$$

Here, $T^{\alpha\beta}$ is the stress tensor, and κ is defined so that

$$\kappa \equiv \sqrt{8\pi G_N}, \tag{1.22}$$

where G_N is Newton's constant. The Planck mass is defined as the inverse of κ so that

$$M_{Pl} \equiv \frac{\sqrt{\hbar c}}{\kappa} \simeq 2.43 \times 10^{18}\ \text{GeV}/c^2. \tag{1.23}$$

Here, the Einstein gauge invariance along with the constraint that no more than two derivatives appear in the Lagrangian or in the equations of motion completely determines the left-hand side of Eq. (1.21). However, the right-hand side of Eq. (1.21) remains undetermined, where a variety of fields including spin 0, spin 1/2, and spin 1 enter. Thus, the stress tensor $T^{\alpha\beta}$ depends on the number of quark and lepton generations and the number of Higgs fields, and on the gauge fields as well as any other matter field that may enter a particle physics Lagrangian. Again, while the Einstein gauge invariance along with the number of derivatives constraint is powerful enough to determine the self-interactions of the gravitational field, it is not powerful enough to fix the number and type of matter and gauge fields that enter on the right-hand side of Eq. (1.21). This is a singular weakness of the Einstein theory, as was realized early on by Einstein himself, who described the gravitational equations as

Similar to a building, one wing of which is made of fine marble, but the other wing of which is built of low grade wood. [15]

String theory in principle resolves this problem, i.e., the problem of determining the right-hand side of Eq. (1.21). For a given string model and for a given vacuum structure, the matter and gauge content can be determined, and thus the right-hand side of Eq. (1.21) is determined. However, there remains the issue of too many possibilities from which one unique case which describes our world must be extracted (for a review of string theory see Green *et al.* [16, 17]).

Another issue that enters into unification is the problem of mass scales, and this idea leads us in a new direction. There is empirical evidence that several mass scales exist in nature. The discovery that the law of reflection symmetry or parity symmetry is violated in weak interactions [18] led to the emergence of a two component equation for the neutrino [19–21] and subsequently to the emergence of the V–A theory of weak interactions [22, 23]. Thus, the interactions of the V–A theory are given by

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} J_\mu(x) J^{\mu\dagger}(x), \tag{1.24}$$

where

$$J_\mu = \ell_\mu + h_\mu. \tag{1.25}$$

Here ℓ_μ is the current containing leptons so that

$$\ell_\mu = \sum_{i=1}^{3} \bar{\nu}_i \gamma_\mu (1-\gamma_5) e_i, \tag{1.26}$$

where the sum runs over the e, the μ and the τ generations and h_μ in Eq. (1.25) is the hadronic current, which is given by

$$h_\mu = \sum_{i=1}^{3} \bar{u}_i V_{ij} (1-\gamma_5) d_j, \tag{1.27}$$

where V_{ij} is the Cabbibo–Kobayashi–Maskawa mixing matrix [24, 25], and G_F appearing in Eq. (1.24) is the Fermi constant and has the value

$$G_F^{-\frac{1}{2}} \simeq 292.8 \text{ GeV}. \tag{1.28}$$

The dimension-six weak interaction operator of Eq. (1.24) is non-renormalizable and points to a more fundamental theory at the scale $G_F^{-1/2}$. Such a theory is the $SU(2)_L \otimes U(1)_Y$ gauge theory of electroweak interactions [26–28]. The spectrum of this theory contains in it $SU(2)_L$ doublets and singlets of quarks and leptons, and gauge bosons of the gauge group $SU(2)_L$ and $U(1)_Y$, i.e.,

$$\begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}_L, \ e^c_{Li} \qquad \text{leptons} \tag{1.29}$$

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \ u^c_{Li}, \ d^c_{Li} \qquad \text{quarks} \tag{1.30}$$

$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad \text{Higgs boson} \tag{1.31}$$

$$A^\mu_\alpha, \ B^\mu \qquad SU(2)_L, U(1)_Y \text{ gauge bosons}, \tag{1.32}$$

where $\alpha = 1{-}3$. The mystery of the scale $G_F^{-\frac{1}{2}}$ is solved as it becomes related to the vacuum expectation value of the Higgs field H^0 such that

$$\begin{aligned} G_F^{-\frac{1}{2}} &= 2^{1/4} v \\ \langle H^0 \rangle &= v/\sqrt{2}. \end{aligned} \tag{1.33}$$

Using Eq. (1.28) one finds

$$v \simeq 246 \text{ GeV}. \tag{1.34}$$

The standard model of electroweak interactions predicts two massive gauge fields, one of which is charged ($W^\pm$) and the other is neutral (Z^0), whose masses are determined in terms of the vacuum expectation value v and the coupling constant g associated with $SU(2)_L$ and the coupling g' associated with $U(1)_Y$ so that

$$M_{W^\pm} = \frac{1}{2}gv,$$
$$M_Z = \frac{v}{2}\sqrt{g^2 + g'^2}. \tag{1.35}$$

The electroweak model has been shown to be renormalizable [29].

While the electroweak model solves the mystery of the Fermi constant, it creates a mystery of its own in terms of the ratio of the coupling constants g'/g or $\sin\theta_W$ (where θ_W is the weak angle) defined by

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}. \tag{1.36}$$

Experimentally,

$$\sin^2\theta_W \simeq 0.23. \tag{1.37}$$

Equation (1.37) has no explanation within the standard model. Indeed, a proper understanding of it requires us to consider an entirely new regime of physics involving a mass scale much larger than $G_F^{-1/2}$. The mystery of this ratio is connected as well with the value of the strong interaction coupling α_s, which we now discuss.

Free quarks are not seen as they occur only as bound states within nucleons. As already discussed, the quarks must carry color, and the simplest hypothesis is that quarks belong to the triplet representation of $SU(3)_C$. One may thus think of expanding the gauge group which describes quarks from $SU(2)_L \otimes U(1)_Y$ to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. An entirely unexpected new feature emerges regarding the group $SU(3)_C$ if one assumes that unlike $SU(2)_L \otimes U(1)_Y$ it is not spontaneously broken but is exactly preserved. In this case, one finds that there would be eight massless gauge bosons which belong to the adjoint representation of the $SU(3)_C$ gauge group, and they can mediate interactions between colored quarks. The interaction of the gluons with quarks in this case is given by

$$\mathcal{L}_{\text{int}} = g_3 \sum_{i,a} \bar{\psi}_i \gamma^\mu T_a A^a_\mu \psi_i, \tag{1.38}$$

where T_a ($a = 1$–8) are the generators of the group $SU(3)_C$. Quite remarkably, non-abelian gauge theories with the appropriate matter content exhibit the phenomenon of asymptotic freedom [30, 31] at large energy scales where the interactions among quarks get weaker and can explain the scaling property seen in high-energy collisions [32]. On the other hand, at low energy the interactions between quarks become stronger, which explains how quarks are confined in nucleons. Indeed, the standard model of electroweak and strong interactions based on the gauge group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \tag{1.39}$$

is one of the most successful models in all of particle physics.

We now turn to another puzzle which relates to the quantum numbers of the quarks and the leptons. The left-handed leptons and the quarks have the following set of $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ quantum numbers:

$$\begin{aligned} &q\left(3,\, 2,\, \frac{1}{6}\right),\ u^c\left(\bar{3},\, 1,\, -\frac{2}{3}\right),\ d^c\left(\bar{3},\, 1,\, \frac{1}{3}\right), \\ &L\left(1,\, 2,\, -\frac{1}{2}\right),\ e^c(1,\, 1,\, 1), \end{aligned} \tag{1.40}$$

where q and L are the doublets of $SU(2)_L$ and u^c, d^c, and e^c are $SU(2)_L$ singlets where we have dropped the subscript L. The assignment of the hypercharges of the particles in the standard model appears strange on the surface. However, there is a interesting property that the hypercharges satisfy, which is

$$\sum_i m_i C_i Y_i = 0, \tag{1.41}$$

where m_i is the multiplicity which counts the number of states in a multiplet and C_i are the number of colors and the sum on i runs over the multiplets in the standard model. This is precisely one of the conditions needed for the cancellation of anomalies. It is known that a cancellation of anomalies (see Chapter 6) among multiplets which couple to gauge fields is needed for a theory to be an acceptable quantum field theory [33, 34]. The question then is, if beyond the standard model there is a larger framework in which an arrangement of colors, iso-spins, and hypercharges as given in Eq. (1.40) can arise in a natural way. Such a framework would need to be more unified, and one possibility is an enlarged gauge group [35,36] which can accommodate the standard model gauge group of Eq. (1.39). A major step in this direction was taken by Pati and Salam [35], who proposed the group $SU(4)_C \times SU(2)_L \times SU(2)_R$ where leptons and quarks are unified with the leptons arising as the fourth color. $SU(4)_C \times SU(2)_L \times SU(2)_R$, however, is a product group, and it is interesting to look for a fully unified group which can accommodate the standard model gauge group. It turns out that $SU(5)$, which is rank four, is the minimal grand unified group [36] which can accommodate the gauge group of Eq. (1.39). In $SU(5)$ one needs a combination of two irreducible representations to accommodate one full generation of the standard model particles, i.e., one needs the irreducible representations $\bar{5} \oplus 10$. The particle content of these is as follows:

$$\bar{5}_L = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu_e \end{pmatrix}_L, \quad 10_L = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}_L. \tag{1.42}$$

The combination of $\bar{5}$ and 10 is anomaly free, and accommodates one full generation of quarks and leptons. Also, since the electric charge is related to iso-spin and hypercharge by the relation Eq. (1.7), one finds that grand unification provides an explanation of the quantization of charge, i.e., that the quarks $(u,\ d)$ have charges $(\frac{2}{3},\ -\frac{1}{3})$ while $(\nu_e,\ e)$ have charges $(0,\ -1)$. This can be viewed as a major triumph of grand unification. One less attractive aspect of $SU(5)$ grand unification is that one generation of quarks and leptons requires two irreducible representations, i.e., $\bar{5} \oplus 10$. It is preferable to have just one irreducible representation where a full generation of quarks and leptons can be accommodated. To do so would require a higher-rank gauge group. The simplest such possibility is the gauge group $SO(10)$ [37, 38], which has a 16-dimensional spinor representation and which decomposes under $SU(5)$ so that

$$16 = 1 \oplus \bar{5} \oplus 10. \tag{1.43}$$

Here, $\bar{5}$ and 10 accommodate a full generation of quarks and leptons as given in Eq. (1.42), and in addition one has a singlet field which we may label as ν^c. Such a field plays a role in giving mass to the neutrino through the so-called see-saw mechanism [39]. One can then generate a neutrino mass matrix, which in the $\nu,\ \nu^c$ basis has the form

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}, \tag{1.44}$$

where m is size the electroweak scale and M is size the grand unified theory (GUT) scale. Diagonalization of this mass matrix generates a light neutrino mass of size m^2/M. Assuming $m = O(M_Z)$ and $M = O(10^{15-16})$ GeV, one finds $m_\nu = O(10^{-2} - 10^{-3})$ eV, which is in the range of desired neutrino masses. Thus, $SO(10)$ is an attractive grand unification group although it has the drawback that currently there are a large number of models that one can build using different Higgs representations to break the grand unification group. In addition, attempts have been made to build models using groups $SU(N)$, $N > 5$, and $SO(N)$ for $N > 10$ as well as exceptional groups. Among the five exceptional groups $G_2,\ F_4,\ E_6,\ E_7$, and E_8, only E_6 has chiral representations, and, further, it contains $SU(5)$ and $SO(10)$ as subgroups so that

$$SU(5) \subset SO(10) \subset E_6. \tag{1.45}$$

One advantage of E_6 is that it allows inclusion of Higgs fields along with quarks and leptons in the same multiplet. But there are also some drawbacks relative to the $SO(10)$ models.

As seen above, an irreducible representation of the grand unification group contains both quarks and leptons in the same multiplet. This means that the quarks can change into leptons by exchange of the so-called lepto-quarks, which are the gauge bosons of the $SU(5)$, $SO(10)$, or E_6 gauge group carrying the quantum numbers of both quarks and leptons. These gauge bosons develop masses by spontaneous breaking of the grand unified symmetry to the symmetry of the standard model gauge group. Let us assume that the GUT symmetry breaks