1 Stress and Strain

Introduction

This book is concerned with the mechanical behavior of materials. The term *mechanical behavior* refers to the response of materials to forces. Under load, a material may either deform or break. The factors that govern a material's resistance to deforming are quite different than those governing its resistance to fracture. The word *strength* may refer either to the stress required to deform a material or to the stress required to cause fracture; therefore, care must be used with the term *strength*.

When a material deforms under small stresses, the deformation may be *elastic*. In this case, when the stress is removed, the material will revert to its original shape. Most of the elastic deformation will recover immediately. There may be, however, some time-dependent shape recovery. This time-dependent elastic behavior is called *anelasticity* or *viscoelasticity*.

Larger stresses may cause *plastic* deformation. After a material undergoes plastic deformation, it will not revert to its original shape when the stress is removed. Usually, high resistance to deformation is desirable so that a part will maintain its shape in service when stressed. However, it is desirable to have materials deform easily when forming them by rolling, extrusion, and so on. Plastic deformation usually occurs as soon as the stress is applied. At high temperatures, however, time-dependent plastic deformation called *creep* may occur.

Fracture is the breaking of a material into two or more pieces. If fracture occurs before much plastic deformation occurs, we say the material is *brittle*. In contrast, if there has been extensive plastic deformation preceding fracture, the material is considered *ductile*. Fracture usually occurs as soon as a critical stress has been reached; however, repeated applications of a somewhat lower stress may cause fracture. This is called *fatigue*.

The amount of deformation that a material undergoes is described by *strain.* The forces acting on a body are described by *stress.* Although the reader should already be familiar with these terms, they will be reviewed in this chapter.

Stress

Stress, σ, is defined as the intensity of force at a point.

$$\sigma = \partial F/\partial A \quad \text{as } \partial A \to 0. \tag{1.1a}$$

If the state of stress is the same everywhere in a body,

$$\sigma = F/A. \tag{1.1b}$$

A *normal stress* (compressive or tensile) is one in which the force is normal to the area on which it acts. With a *shear stress,* the force is parallel to the area on which it acts. Two subscripts are required to define a stress. The first subscript denotes the normal to the plane on which the force acts, and the second subscript identifies the direction of the force.* For example, a tensile stress in the x-direction is denoted by σ_{xx} indicating that the force is in the x-direction and it acts on a plane normal to x. For a shear stress, σ_{xy}, a force in the y-direction acts on a plane normal to x.

Because stresses involve both forces and areas, they are not vector quantities. Nine components of stress are needed to fully describe a state of stress at a point, as shown in Figure 1.1. The stress component, $\sigma_{yy} = F_y/A_y$, describes the tensile stress in the y-direction. The stress component, $\sigma_{zy} = F_y/A_z$, is the shear stress caused by a shear force in the y-direction acting on a plane normal to z.

Repeated subscripts denote normal stresses (e.g., σ_{xx}, σ_{yy}), whereas mixed subscripts denote shear stresses (e.g., σ_{xy}, σ_{zx}). In *tensor* notation, the state of stress is expressed as

$$\sigma_{ij} = \begin{vmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix} \tag{1.2}$$

where i and j are iterated over x, y, and z. Except where tensor notation is required, it is often simpler to use a single subscript for a normal stress and to denote a shear stress by τ:

$$\sigma_x = \sigma_{xx}, \quad \text{and} \quad \tau_{xy} = \sigma_{xy}. \tag{1.3}$$

A stress component expressed along one set of axes may be expressed along another set of axes. Consider the case in Figure 1.2. The body is subjected to a stress

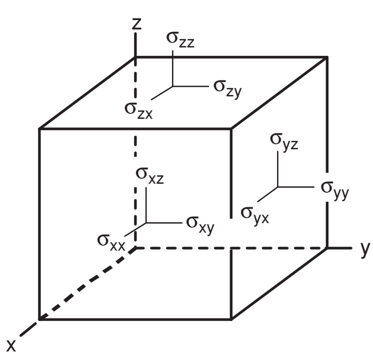

Figure 1.1. Nine components of stress acting on an infinitesimal element. Normal stress components are σ_{xx}, σ_{yy}, and σ_{zz}. Shear stress components are σ_{yz}, σ_{zx}, σ_{xy}. σ_{zy}, σ_{xz}, and σ_{yx}.

* Use of the opposite convention should not cause confusion because $\sigma_{ij} = \sigma_{ji}$.

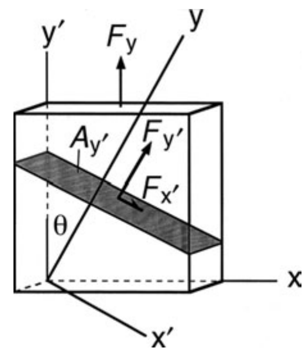

Figure 1.2. Stresses acting on an area, $A_{y'}$, under a normal force, F_y. The normal stress, $\sigma_{y'y'} = F_{y'}/A_{y'} = F_y \cos\theta/(A_y/\cos\theta) = \sigma_{yy}\cos^2\theta$. The shear stress, $\tau_{y'x'} = F_x/A_{y'} = F_y \sin\theta/(A_{yx}/\cos\theta) = \sigma_{yy}\cos\theta\sin\theta$.

$\sigma_{yy} = F_y/A_y$. It is possible to calculate the stress acting on a plane whose normal, y', is at an angle θ to y. The normal force acting on the plane is $F_{y'} = F_y \cos\theta$, and the area normal to y′ is $A_y/\cos\theta$, so

$$\sigma_{y'} = \sigma_{y'y'} = F_{y'}/A_{y'} = (F_y \cos\theta)/(A_y/\cos\theta) = \sigma_y \cos^2\theta. \quad (1.4a)$$

Similarly, the shear stress on this plane acting in the x′-direction, $\tau_{y'x'}(= \sigma_{y'x'})$, is given by

$$\tau_{y'x'} = \sigma_{y'x'} = F_{x'}/A_{y'} = (F_y \sin\theta)/(A_y/\cos\theta) = \sigma_y \cos\theta\sin\theta. \quad (1.4b)$$

Note: The transformation requires the product of two cosine and/or sine terms.

Sign Convention

When we write $\sigma_{ij} = F_i/A_j$, the term σ_{ij} is positive if i and j are either both positive or both negative. However, the stress component is negative for a combination of i and j in which one is positive and the other is negative. For example, in Figure 1.3, the terms σ_{xx} are positive on both sides of the element because both the force and normal to the area are negative on the left and positive on the right. The stress, τ_{yx}, is negative because on the top surface y is positive and x-direction force is negative, and on the bottom surface, x-direction force is positive and the normal to the area, y, is negative. Similarly, τ_{xy} is negative.

Pairs of shear stress terms with reversed subscripts are always equal. A moment balance requires that $\tau_{ij} = \tau_{ji}$. If they were not, the element would undergo an infinite rotational acceleration (Figure 1.4). For example, $\tau_{yx} = \tau_{xy}$. Therefore, we

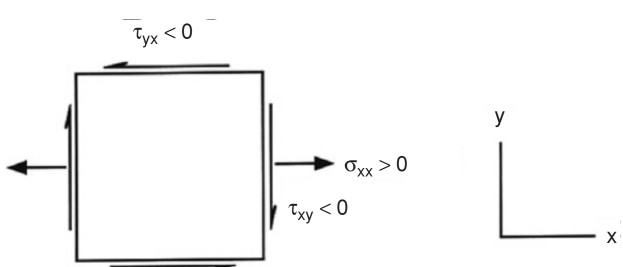

Figure 1.3. The normal stress, σ_{xx}, is positive because the direction of the force, F_x, and the normal to the plane are either both positive (*right*) or both negative (*left*). The shear stresses, τ_{xy} and τ_{yx}, are negative because the direction of the force and the normal to the plane have opposite signs.

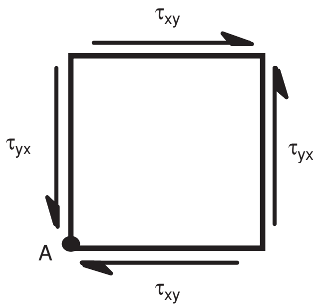

Figure 1.4. An infinitesimal element under shear stresses, τ_{xy} and τ_{yx}. A moment balance about A requires that $\tau_{xy} = \tau_{yx}$.

can write, in general, that $\Sigma M_A = \tau_{yx} = \tau_{xy} = 0$, so

$$\sigma_{ij} = \sigma_{ji,} \quad \text{or} \quad \tau_{ij} = \tau_{ji}. \tag{1.5}$$

This makes its stress tensor symmetric about the diagonal.

Transformation of Axes

Frequently, we must change the axis system on which a stress state is expressed. For example, we may want to find the shear stress on a slip system from the external stresses acting on a crystal. Another example is finding the normal stress across a glued joint in a tube subjected to tension and torsion. In general, a stress state expressed along one set of orthogonal axes (e.g., m, n, and p) may be expressed along a different set of orthogonal axes (e.g., i, j, and k). The general form of the transformation is

$$\sigma_{ij} = \sum_{n=1}^{3}\sum_{m=1}^{3} \ell_{im}\ell_{jn}\sigma_{mn}. \tag{1.6}$$

The term, ℓ_{im}, is the cosine of the angle between the i and m axes, and ℓ_{jn} is the cosine of the angle between the j and n axes. The summations are over the three possible values of m and n, namely, m, n, and p. This is often written as

$$\sigma_{ij} = \ell_{im}\ell_{jn}\sigma_{mn}, \tag{1.7}$$

with the summation implied. The stresses in the x, y, z coordinate system in Figure 1.5 may be transformed onto the x′, y′, z′ coordinate system by

$$\begin{aligned}\sigma_{x'x'} = {} & \ell_{x'x}\ell_{x'x}\sigma_{xx} + \ell_{x'y}\ell_{x'x}\sigma_{yx} + \ell_{x'z}\ell_{x'x}\sigma_{zx} \\ & + \ell_{x'x}\ell_{x'y}\sigma_{xy} + \ell_{x'y}\ell_{x'y}\sigma_{yy} + \ell_{x'z}\ell_{x'y}\sigma_{zy} \\ & + \ell_{x'x}\ell_{x'z}\sigma_{xz} + \ell_{x'y}\ell_{x'z}\sigma_{yz} + \ell_{x'z}\ell_{x'z}\sigma_{zz}\end{aligned} \tag{1.8a}$$

and

$$\sigma_{x'y'} = \ell_{x'x}\ell_{y'x}\sigma_{xx} + \ell_{x'y}\ell_{y'x}\sigma_{yx} + \ell_{x'z}\ell_{y'x}\sigma_{zx}$$

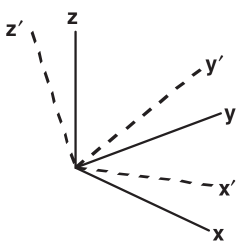

Figure 1.5. Two orthogonal coordinate systems, x, y, and z and x′, y′, and z′. The stress state may be expressed in terms of either.

$$+\ell_{x'x}\ell_{y'y}\sigma_{xy}+\ell_{x'y}\ell_{y'y}\sigma_{yy}+\ell_{x'z}\ell_{y'y}\sigma_{zy}$$
$$+\ell_{x'x}\ell_{y'z}\sigma_{xz}+\ell_{x'y}\ell_{y'z}\sigma_{yz}+\ell_{x'z}\ell_{y'z}\sigma_{zz}. \quad (1.8b)$$

These equations may be simplified with the notation in equation (1.3) using equation (1.5),

$$\sigma_{x'}=\ell_{x'x}2\sigma_x+\ell_{x'y}2\sigma y+\ell_{x'z}2\sigma_z$$
$$+2\ell_{x'y}\ell_{x'z}\tau_{yz}+2\ell_{x'z}\ell_{x'x}\tau_{zx}+2\ell_{x'x}\ell_{x'y}\tau_{xy} \quad (1.9a)$$

and

$$\tau_{x'z'}=\ell_{x'x}\ell_{y'x}\sigma_{xx}+\ell_{x'y}\ell_{y'y}\sigma_{yy}+\ell_{x'z}\ell_{y'z}\sigma_{zz}$$
$$+(\ell_{x'y}\ell 1_{y'z}+\ell_{x'z}\ell_{y'y})\tau_{yz}+(\ell_{x'z}\ell_{y'x}+\ell_{x'x}\ell_{y'z)}\tau_{zx}$$
$$+(\ell_{x'x}\ell_{y'y}+\ell_{x'y}\ell_{y'x})\tau_{xy}. \quad (1.9b)$$

Now reconsider the transformation in Figure 1.2. Using equations (1.9a) and (1.9b), with σ_{yy} as the only finite term on the x, y, z axis system,

$$\sigma y'=\ell_{y'y}^{2}\sigma_{yy}=\sigma_y\cos^2\theta \quad \text{and} \quad \tau_{x'y'}=\ell_{x'y}\ell_{y'y}\sigma_{yy}=\sigma_y\cos\theta\sin\theta \quad (1.10)$$

in agreement with equations 1.4a and 1.4b. These equations can be used together with Miller indices for planes and direction indices for crystals. The reader that is not familiar with these is referred to Appendix I.

EXAMPLE PROBLEM 1.1: A cubic crystal is loaded with a tensile stress of 2.8 MPa applied along the [210] direction, as shown in Figure 1.6. Find the shear stress on the (111) plane in the [10$\bar{1}$] direction.

Solution: In a cubic crystal, the normal to a plane has the same indices as the plane, so the normal to (111) is [111]. Also, in a cubic crystal, the cosine of the angle between two directions is given by the dot product of unit vectors in those

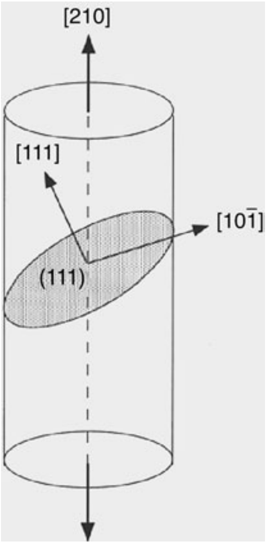

Figure 1.6. A crystal stressed in tension along [210] showing the (111) slip plane and the [10$\bar{1}$] slip direction.

directions. For example, the cosine of the angle between $[u_1v_1w_1]$ and $[u_2v_2w_2]$ is equal to $(u_1u_2 + v_1v_2 + w_1w_2)/[(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)]^{1/2}$. Designating [210] as x, $[10\bar{1}]$ as d, and [111] as n, $\tau_{nd} = \ell_{nx}\ell_{dx}\sigma_{xx} = \{(2\cdot1 + 1\cdot1 + 0)/\sqrt{[(2^2 + 1^2 + 0)(1^2 + 1^2 + 1^2)]}\}\cdot\{(2\cdot1 + 1\cdot0 + 0\cdot0)/\sqrt{[(2^2 + 1^2 + 0)(1^2 + 0 + 1^2)]}\}2.8\text{ MPa} = 2.8(6/5)\sqrt{6} = 1.372$ MPa.

Principal Stresses

It is always possible to find a set of axes (1, 2, 3) along which the shear stress components vanish. In this case, the normal stresses, σ_1, σ_2, and σ_3, are called *principal stresses*, and the 1, 2, and 3 axes are the *principal stress axes*. The magnitudes of the principal stresses, σ_p, are the three roots of

$$\sigma_p^3 - I_1\sigma_p^2 - I_2\sigma_p - I_3 = 0, \tag{1.11}$$

where

$$\begin{aligned} I_1 &= \sigma_{xx} + \sigma_{yy} + \sigma_{zz},\\ I_2 &= \sigma_{yz}^2 + \sigma_{zx}^2 + \sigma_{xy}^2 - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} - \sigma_{xx}\sigma_{yy},\\ I_3 &= \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{yz}\sigma_{zx}\sigma_{xy} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{zx}^2 - \sigma_{zz}\sigma_{xy}^2. \end{aligned} \tag{1.12}$$

The first invariant, $I_1 = -p/3$, where p is the pressure. I_1, I_2, and I_3 are independent of the orientation of the axes and are therefore called *stress invariants*. In terms of the principal stresses, the invariants are

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3,\\ I_2 &= -\sigma_{22}\sigma_{33} - \sigma_{33}\sigma_{11} - \sigma_{11}\sigma_{22},\\ I_3 &= \sigma_{11}\sigma_{22}\sigma_{33}. \end{aligned} \tag{1.13}$$

EXAMPLE PROBLEM 1.2: Find the principal stresses in a body under the stress state, $\sigma_x = 10$, $\sigma_y = 8$, $\sigma_z = -5$, $\tau_{yz} = \tau_{zy} = 5$, $\tau_{zx} = \tau_{xz} = -4$, and $\tau_{xy} = \tau_{yx} = -8$, where all stresses are in MPa.

Solution: Using equation (1.13), $I_1 = 10 + 8 - 5 = 13$, $I_2 = 5^2 + (-4)^2 + (-8)^2 - 8(-5) - (-5)10 - 10\cdot8 = 115$, $I_3 = 10\times8(-5) + 2\times5(-4)(-8) - 10\times5^2 - 8(-4)^2 - (-5)(-8)^2 = -138$.

Solving equation (1.11), $\sigma_p^3 - 13\sigma_p^2 - 115\sigma_p + 138 = 0$, $\sigma_p = 1.079$, 18.72, –6.82.

Mohr's Stress Circles

In the special case where there are no shear stresses acting on one of the reference planes (e.g., $\tau_{zy} = \tau_{zx} = 0$), the normal to that plane, z, is a direction of principal stress, and the other two principal stress directions lie in the plane. This is illustrated in Figure 1.7. For these conditions, $\ell_{x'z} = \ell_{y'z} = 0$, $\tau_{zy} = \tau_{zx} = 0$, $\ell_{x'x} = \ell_{y'y} = $ cosf, and $\ell_{x'y} = -\ell_{y'x} = \sin\phi$. The variation of the shear stress component, $\tau_{x'y'}$, can be found by substituting these conditions into the stress transformation equation (1.8b). Substituting $\ell_{x'z} = -\ell_{y'z} = 0$,

$$\tau_{x'y'} = \cos\phi\,\sin\phi(-\sigma_{xx} + \sigma_{yy}) + (\cos^2\phi - \sin^2\phi)\tau_{xy}. \tag{1.14a}$$

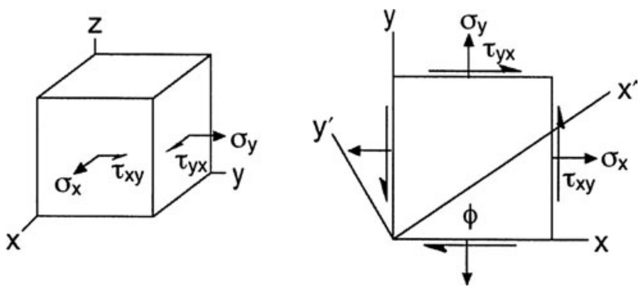

Figure 1.7. Stress state to which Mohr's circle treatment applies. Two shear stresses, τ_{yz} and τ_{zx}, are zero.

Similar substitution into the expressions for $\sigma_{x'}$ and $\sigma_{y'}$ results in

$$\sigma_{x'} = \cos^2\phi\sigma_x + \sin^2\phi\sigma_y + 2\cos\phi\sin\phi\tau_{xy}. \tag{1.14b}$$

and

$$\sigma_{y'} = \sin^2\phi\sigma_x + \cos^2\phi\sigma_y + 2\cos\phi\sin\phi\tau_{xy}. \tag{1.14c}$$

These can be simplified by substituting the trigonometric identities, $\sin 2\phi = 2\sin\phi\cos\phi$ and $\cos 2\phi = \cos^2\phi - \sin^2\phi$,

$$\tau_{x'y'} = -[(\sigma_x - \sigma_y)/2]\sin 2\phi + \tau_{xy}\cos 2\phi \tag{1.15a}$$

$$\sigma_{x'} = (\sigma_x + \sigma_y)/2 + [\sigma_x - \sigma_y)/2]\cos 2\phi + \tau_{xy}\sin 2\phi. \tag{1.15b}$$

and

$$\sigma_{y'} = (\sigma_x + \sigma_y)/2 - [\sigma_x - \sigma_y)/2]\cos 2\phi + \tau_{xy}\sin 2\phi. \tag{1.15c}$$

Setting $\tau_{x'y'} = 0$ in equation 1.15a, becomes the angle, θ, between the principal stresses axes and the x and y axes. See Figure 1.8. $\tau_{x'y'} = 0 = \sin 2\theta(\sigma_x - \sigma_y)/2 + \cos 2\theta\tau_{xy}$ or

$$\tan 2\theta = \tau_{xy}/[(\sigma_x - \sigma_y)/2]. \tag{1.16}$$

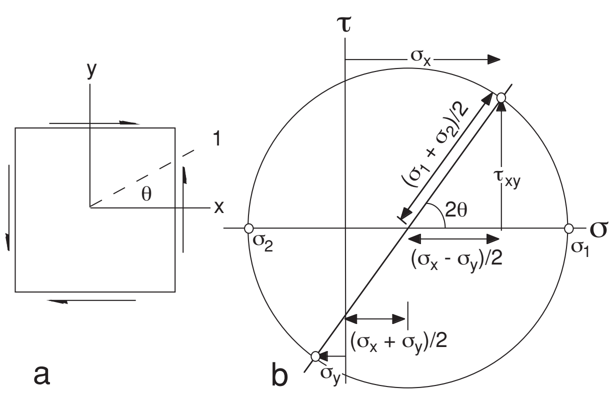

Figure 1.8. Mohr's circles for stresses showing the stresses in the x-y plane. *Note:* The 1-axis is rotated counterclockwise from the x-axis in real space (a), whereas in the Mohr's circle diagram, the 1-axis is rotated clockwise from the x axis (b).

The principal stresses, σ_1 and σ_2, are the values of $\sigma_{x'}$ and $\sigma_{y'}$ for this value of ϕ,

$$\begin{aligned}\sigma_{1,2} &= (\sigma_x + \sigma_y)/2 \pm [\sigma_x - \sigma_y)/2]\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{or}\\ \sigma_{1,2} &= (\sigma_x + \sigma_y)/2 \pm (1/2)[(\sigma_x - \sigma_y)^2 + 4\tau_{xy^2}]^{1/2}\end{aligned} \tag{1.17}$$

A Mohr's circle diagram is a graphical representation of equations (1.16) and (1.17). It plots as a circle with a radius $(\sigma_1 - \sigma_2)/2$ centered at

$$(\sigma_1 + \sigma_2)/2 = (\sigma_x + \sigma_y)/2, \tag{1.17a}$$

as shown in Figure 1.8. The normal stress components, σ, are represented on the ordinate and the shear stress components, τ, on the abscissa. Consider the triangle in Figure 1.8b. Using the Pythagorean theorem, the hypotenuse,

$$(\sigma_1 - \sigma_2)/2 = \left\{[(\sigma_x + \sigma_y)/2]^2 + \tau_{xy}^2\right\},^{1/2} \tag{1.17b}$$

and

$$\tan(2\theta) = [\tau_{xy}/[(\sigma_x + \sigma_y)/2]. \tag{1.17c}$$

The full three-dimensional stress state may be represented by three Mohr's circles (Figure 1.9).

The three principal stresses, σ_1, σ_2, and σ_3, are plotted on the horizontal axis. The circles connecting these represent the stresses in the 1–2, 2–3, and 1–3 planes. The largest shear stress may be either $(\sigma_1 - \sigma_2)/2$, $(\sigma_2 - \sigma_3)/2$, or $(\sigma_1 - \sigma_3)/2$.

EXAMPLE PROBLEM 1.3: A body is loaded under stresses, $\sigma_x = 150$ MPa, $\sigma_y = 60$ MPa, $\tau_{xy} = 20$ MPa, $\sigma_z = \tau_{yz} = \tau_{zx} = 0$. Find the three principal stresses, sketch the three-dimensional Mohr's circle diagram for this stress state, and find the largest shear stress in the body.

Solution: σ_1, $\sigma_2 = (\sigma_x + \sigma_y)/2 \pm \{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2\}^{1/2} = 154.2$, 55.8 MPa, $\sigma_3 = \sigma_z = 0$. Figure 1.10 is the Mohr's circle diagram. Note that the largest shear stress, $\tau_{max} = (\sigma_1 - \sigma_3)/2 = 77.1$ MPa, is not in the 1–2 plane.

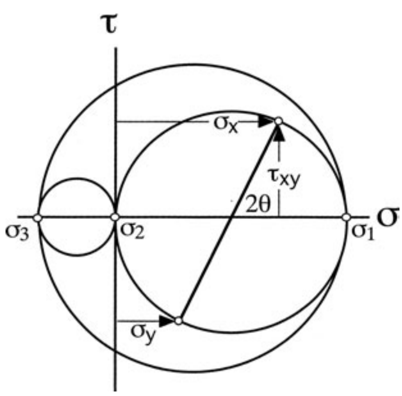

Figure 1.9. Three Mohr's circles representing a stress state in three dimensions. The three circles represent the stress states in the 2–3, 3–1, and 1–2 planes.

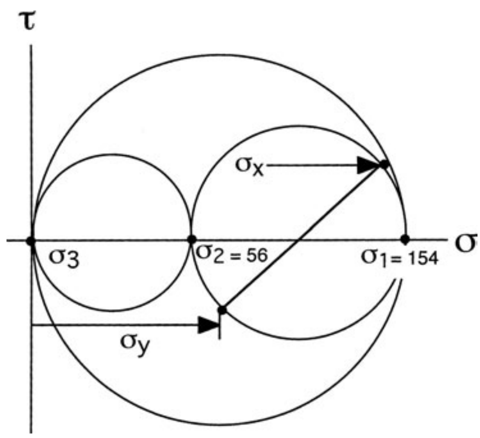

Figure 1.10. Mohr's circles for example problem 1.3.

Strains

An infinitesimal normal strain is defined as strain by the change of length, L, of a line:

$$d\varepsilon = dL/L. \tag{1.18}$$

Integrating from the initial length, L_o, to the current length, L,

$$\varepsilon = \int dL/L = \ln(L/L_o). \tag{1.19}$$

This finite form is called *true strain* (or *natural strain, or logarithmic strain*). Alternatively, *engineering* or *nominal strain, e*, is defined as

$$e = \Delta L/L_o. \tag{1.20}$$

If the strains are small, then engineering and true strains are nearly equal. Expressing $\varepsilon = \ln(L/L_o) = \ln(1+e)$ as a series expansion, $\varepsilon = e - e^2/2 + e^3/3! \ldots$ so as $e \to 0$, $\varepsilon \to e$. This is illustrated in example problem 1.4.

EXAMPLE PROBLEM 1.4: Calculate the ratio of e/ε for several values of e.

Solution: $e/\varepsilon = e/\ln(1+e)$. Evaluating:

for $e = 0.001$, $e/\varepsilon = 1.0005$;
for $e = 0.01$, $e/\varepsilon = 1.005$;
for $e = 0.02$, $e/\varepsilon = 1.010$;
for $e = 0.05$, $e/\varepsilon = 1.025$;
for $e = 0.10$, $e/\varepsilon = 1.049$;
for $e = 0.20$, $e/\varepsilon = 1.097$;
for $e = 0.50$, $e/\varepsilon = 1.233$.

Note that the difference e and ε between is about 1% for $e < 0.02$.

There are several reasons that true strains are more convenient than engineering strains.

1. True strains for equivalent amounts of deformation in tension and compression are equal except for sign.

2. True strains are additive. For a deformation consisting of several steps, the overall strain is the sum of the strains in each step.
3. The volume change is related to the sum of the three normal strains. For constant volume, $\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$.

These statements are not true for engineering strains, as illustrated in the following examples.

EXAMPLE PROBLEM 1.5: An element 1 cm long is extended to twice its initial length (2 cm) and then compressed to its initial length (1 cm).

A. Find true strains for the extension and compression.
B. Find engineering strains for the extension and compression.

Solution:

A. During the extension, $\varepsilon = \ln(L/L_o) = \ln 2 = 0.693$, and during the compression,

$$\varepsilon = \ln(L/L_o) = \ln(1/2) = -0.693.$$

B. During the extension, $e = \Delta L/L_o = 1/1 = 1.0$, and during the compression,

$$e = \Delta L/L_o = -1/2 = -0.5.$$

Note that with engineering strains, the magnitude of strain to reverse the shape change is different.

EXAMPLE PROBLEM 1.6: A bar 10 cm long is elongated by (1) drawing to 15 cm, and then (2) drawing to 20 cm.

A. Calculate the engineering strains for the two steps, and compare the sum of these with the engineering strain calculated for the overall deformation.
B. Repeat the calculation with true strains.

Solution:

A. For step 1, $e_1 = 5/10 = 0.5$; for step 2, $e_2 = 5/15 = 0.333$. The sum of these is 0.833, which is less than the overall strain, $e_{tot} = 10/10 = 1.00$
B. For step 1, $\varepsilon_1 = \ln(15/10) = 0.4055$; for step 2, $\varepsilon_1 = \ln(20/15) = 0.2877$. The sum is 0.6931, and the overall strain is $\varepsilon_{tot} = \ln(15/10) + \ln(20/15) = \ln(20/10) = 0.6931$.

EXAMPLE PROBLEM 1.7: A block of initial dimensions L_{x0}, L_{y0}, L_{z0} is deformed so that the new dimensions are L_x, L_y, L_z. Express the volume strain, $\ln(V/V_o)$, in terms of the three true strains, ε_x, ε_y, ε_z.

Solution: $V/V_o = L_x L_y L_z/(L_{xo} L_{yo} L_{zo})$, so

$$\ln(V/V_o) = \ln(L_x/L_{xo}) + \ln(L_y/L_{yo}) + \ln(L_z/L_{zo}) = \varepsilon_x + \varepsilon_y + \varepsilon_z.$$

Note that if there is no volume change, ($\ln(V/V_o) = 0$), the sum of the normal strains

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = 0.$$