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An Introduction to Random Matrices

The theory of random matrices plays an important role in many areas of pure mathematics and employs a variety of sophisticated mathematical tools (analytical, probabilistic and combinatorial). This diverse array of tools, while attesting to the vitality of the field, presents several formidable obstacles to the newcomer, and even the expert probabilist.

This rigorous introduction to the basic theory is sufficiently self-contained to be accessible to graduate students in mathematics or related sciences, who have mastered probability theory at the graduate level, but have not necessarily been exposed to advanced notions of functional analysis, algebra or geometry. Useful background material is collected in the appendices and exercises are also included throughout to test the reader's understanding. Enumerative techniques, stochastic analysis, large deviations, concentration inequalities, disintegration and Lie algebras all are introduced in the text, which will enable readers to approach the research literature with confidence.

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To Meredith, Benoit and Naomi

Contents

Preface

The study of random matrices, and in particular the properties of their eigenvalues, has emerged from the applications, first in data analysis and later as statistical models for heavy-nuclei atoms. Thus, the field of random matrices owes its existence to applications. Over the years, however, it became clear that models related to random matrices play an important role in areas of pure mathematics. Moreover, the tools used in the study of random matrices came themselves from different and seemingly unrelated branches of mathematics.

At this point in time, the topic has evolved enough that the newcomer, especially if coming from the field of probability theory, faces a formidable and somewhat confusing task in trying to access the research literature. Furthermore, the background expected of such a newcomer is diverse, and often has to be supplemented before a serious study of random matrices can begin.

We believe that many parts of the field of random matrices are now developed enough to enable one to expose the basic ideas in a systematic and coherent way. Indeed, such a treatise, geared toward theoretical physicists, has existed for some time, in the form of Mehta's superb book [Meh91]. Our goal in writing this book has been to present a rigorous introduction to the basic theory of random matrices, including free probability, that is sufficiently self-contained to be accessible to graduate students in mathematics or related sciences who have mastered probability theory at the graduate level, but have not necessarily been exposed to advanced notions of functional analysis, algebra or geometry. Along the way, enough techniques are introduced that we hope will allow readers to continue their journey into the current research literature.

This project started as notes for a class on random matrices that two of us (G. A. and O. Z.) taught in the University of Minnesota in the fall of 2003, and notes for a course in the probability summer school in St. Flour taught by A. G. in the summer of 2006. The comments of participants in these courses, and in particular A. Bandyopadhyay, H. Dong, K. Hoffman-Credner, A. Klenke, D. Stanton and P.M. Zamfir, were extremely useful. As these notes evolved, we taught from them again at the University of Minnesota, the University of California at Berkeley, the Technion and the Weizmann Institute, and received more much appreciated feedback from the participants in those courses. Finally, when expanding and refining these course notes, we have profited from the comments and questions of many colleagues. We would like in particular to thank G. Ben Arous, F. Benaych-Georges, P. Biane, P. Deift, A. Dembo, P. Diaconis, U. Haagerup, V. Jones, M. Krishnapur, Y. Peres, R. Pinsky, G. Pisier, B. Rider, D. Shlyakhtenko, B. Solel, A. Soshnikov, R. Speicher, T. Suidan, C. Tracy, B. Virag and D. Voiculescu for their suggestions, corrections and patience in answering our questions or explaining their work to us. Of course, any remaining mistakes and unclear passages are fully our responsibility.

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