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Preface

This book treats *isomorphism theory* – that branch of ergodic theory dealing with the question of when two measure-preserving systems are, in a certain sense, essentially equivalent. Although these topics have received fair treatment in several books, we think that the time is right for a fresh perspective. Indeed, with ergodic theory becoming more fashionable in its connections with number theory and additive combinatorics, yet also more abstract and structure-laden, it is interesting to observe the extent to which progress in its original concerns, classification of measure-preserving systems up to isomorphism, was achieved via combinatorial/probabilistic reasoning. Our hope is that the ergodic theory revival currently underway will find its way to isomorphism theory, and revitalize it as well.

We have also attempted to write a book that teaches general mathematical thinking in a unique manner. Most graduate level textbooks in pure mathematics provide detailed proofs of theorems followed by exercises. We[1] have tried to write this book in such a way as to make the proofs of the theorems themselves the exercises. Optional details, of which readers may want more or less, may be relegated to footnotes or to sections labeled “Remark” or “Comment”.

Indeed, proofs of major theorems are generally presented twice; once labeled “Idea of proof”, in which the reader is called on to flesh out the argument from a very basic outline, then again with the label “Sketch of proof”, in which more details are given. We consider it important that the reader attempt to work through the “Idea" section before or instead of the “Sketch". This reading strategy benefits:

(a) the serious beginner wishing to work out the proofs on their own, with helpful guidance from the book;
(b) the student wanting a basic overview of ergodic theory, who doesn’t want to be overburdened with notation while trying to understand the ideas;
(c) the professional mathematician who is familiar with the material, but wants a quick review or different perspective on it.

[1] The first author has circulated a rough precursor to this book as an MS Word document on the Internet for a number of years; the pedagogic philosophy employed here (and all original proofs of major theorems) are due to him. R.M.

This book differs from other books of its type not only in the careful compartmentalization of detail; much in our approach to classical theorems (those of Shannon–McMillan–Breiman and Ornstein, for example) would be considered unorthodox, however presented. Indeed, our aim is not to present "distinctively modern", "slick" or "book" proofs. Our aim is to present *our* proofs, stumbled upon by active engagement with the subject matter. Our hope is that readers will follow our example, and come to favor their own proofs as well.