

CHAPTER

1

Introduction

This text concerns the basic elementary physics of *plasmas*, which are a special class of gases made up of a large number of electrons and ionized atoms and molecules, in addition to neutral atoms and molecules as are present in a normal (non-ionized) gas. The most important distinction between a plasma and a normal gas is the fact that mutual Coulomb interactions between charged particles are important in the dynamics of a plasma and cannot be disregarded. When a neutral gas is raised to a sufficiently high temperature, or when it is subjected to electric fields of sufficient intensity, the atoms and molecules of the gas may become ionized, electrons being stripped off by collisions as a result of the heightened thermal agitation of the particles. Ionization in gases can also be produced as a result of illumination with ultraviolet light or X-rays, by bombarding the substance with energetic electrons and ions, or in other ways. When a gas is ionized, even to a rather small degree, its dynamical behavior is typically dominated by the electromagnetic forces acting on the free ions and electrons, and it begins to conduct electricity. The charged particles in such an ionized gas interact with electromagnetic fields, and the organized motions of these charge carriers (e.g., electric currents, fluctuations in charge density) can in turn produce electromagnetic fields. The ability of an ionized gas to sustain electric current is particularly important in the presence of a magnetic field. The presence of mobile charged particles in a magnetic field yields a Lorentz force $q\mathbf{v} \times \mathbf{B}$. When applied to a collection of particles this force leads to an electromagnetic body force $\mathbf{J} \times \mathbf{B}$ which can dominate the gas dynamics. As a result, the most novel and spectacular behavior of plasmas is exhibited in the context of their interaction with a magnetic field.

During the 1920s, I. Langmuir and colleagues first showed that characteristic electrical oscillations of very high frequency can exist

in an ionized gas that is neutral or quasi-neutral, and introduced the terms *plasma* and *plasma oscillations*,¹ in recognition of the fact that these oscillations resembled those of jelly-like substances [1, 2]. When subjected to a static electric field, the charge carriers in an ionized gas rapidly redistribute themselves in such a way that most of the gas is shielded from the field, in a manner quite similar to the redistribution of charge which occurs within a metallic conductor placed in an electric field, resulting in zero electric field everywhere inside. Langmuir gave the name “plasma” specifically to the relatively field-free regions of the ionized gas, which are not influenced by the boundaries. Near the boundaries, typically metallic surfaces held at prescribed potentials, strong space-charge fields exist in a transition region Langmuir termed the *plasma sheath*. The sheath region has properties that differ from the plasma, since the motions of charged particles within the sheath are predominantly influenced by the potential of the boundary. The particles in the sheath form an electrical screen between the plasma and the boundary. We will find later that the screening distance is a function of the density of charged particles and of their temperature.

The plasma medium is often referred to as the fourth state of matter, since it has properties profoundly different from those of the gaseous, liquid, and solid states. All states of matter represent different degrees of organization, corresponding to certain values of binding energy. In the solid state, the important quantity is the binding energy of molecules in a crystal. If the average kinetic energy of a molecule exceeds the binding energy (typically a fraction of an electron volt), the crystal structure breaks up, either into a liquid or directly into a gas (e.g., iodine). Similarly, a certain minimum kinetic energy is required in order to break the bonds of the van der Waals forces in order for a liquid to change into a gas. In order for matter to make the transition to its fourth state and exist as a plasma, the kinetic energy per plasma particle must exceed the ionizing potential of atoms (typically a few electron volts). Thus, the state of matter is basically determined by the average kinetic energy per particle. Using water as a convenient example, we note that at low temperatures the bond between the H₂O molecules holds them tightly together against the low energy of molecular motion, so that the matter is in the solid state (ice). At room temperature, the

¹ The word “plasma” first appeared as a scientific term in 1839 when the Czech biologist J. Purkyně coined the term “protoplasma” to describe the jelly-like medium containing a large number of floating particles which make up the nuclei of the cells. The word “plasma” thus means a mold or form, and is also used for the liquid part of blood in which corpuscles are suspended.

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increased molecular energy permits more widespread movements and currents of molecular motion, and we have the liquid state (water). Since the particle motions are random, not all particles have the same energy, with the more energetic ones escaping from the liquid surface to form a vapor above it. As the temperature of the water is further increased, a larger fraction of molecules escapes, until the whole substance is in the gaseous phase (steam). If steam is subjected to further thermal heating, illumination by UV or X-rays, or bombardment by energetic particles, it becomes ionized (plasma).

Although by far most of the Universe is ionized and is therefore in a plasma state, on our planet plasmas have to be generated by special processes and under special conditions. While we live in a bubble of essentially non-ionized gas in the midst of an otherwise ionized environment, examples of partially ionized gases or plasmas, including fire, lightning, and the aurora borealis have long been part of our natural environment. It is in this connection that early natural philosophers held that the material Universe is built of four “roots,” earth, water, air, and fire, curiously resembling our modern terminology of solid, liquid, gas, and plasma states of matter. A transient plasma exists in the Earth’s atmosphere every time a lightning stroke occurs, but is clearly not very much at home and is short-lived. Early work on electrical discharges included generation of electric sparks by rubbing a large rotating sphere of sulphur against a cloth [3], production of sparks by harnessing atmospheric electricity in rather hazardous experiments [4], and studies of dust patterns left by a spark discharge passing through the surface of an insulator [5]. However, it was only when electrical and vacuum techniques were developed to the point where long-lived and relatively stable electrical discharges were available that the physics of ionized gases emerged as a field of study. In 1879, W. Crookes published the results of his investigations of discharges at low pressure and remarked: “The phenomena in these exhausted tubes reveal to physical science a new world, a world where matter may exist in a fourth state . . .” [6]. A rich period of discoveries followed, leading to Langmuir’s coining of the word “plasma” in 1929, and continuing into the present as a most fascinating branch of physics.

Although a plasma is often considered to be the fourth state of matter, it has many properties in common with the gaseous state. At the same time, the plasma is an ionized gas in which the long range of Coulomb forces gives rise to collective interaction effects, resembling a fluid with a density higher than that of a gas. In its most general sense, a plasma is any state of matter which contains enough free, charged particles for its dynamical behavior to be dominated by electromagnetic forces. Plasma physics therefore

encompasses the solid state, since electrons in metals and semi-conductors fall into this category [7]. However, the redistribution of charge and the screening of the inner regions of a metal occur extremely quickly (typically $\sim 10^{-19}$ s) as a result of the very high density of free charges. Most applications of plasma physics are concerned with ionized gases. It turns out that a very low degree of ionization is sufficient for a gas to exhibit electromagnetic properties and behave as a plasma: a gas achieves an electrical conductivity of about half its possible maximum at about 0.1% ionization and has a conductivity nearly equal to that of a fully ionized gas at 1% ionization. The degree of ionization can be defined as the ratio $N_e/(N_e + N_n)$, where N_e is the electron density and N_n is the density of neutral molecules. (Since most plasmas are macroscopically neutral, as we will see later, the density of positive ions is equal to the density of electrons, i.e., $N_i = N_e$.) As an example, the degree of ionization in a fluorescent tube is $\sim 10^{-5}$, with $N_n \simeq 10^{22} \text{ m}^{-3}$ and $N_e \simeq 10^{17} \text{ m}^{-3}$. Typically, a gas is considered to be a weakly (strongly) ionized gas if the degree of ionization is less than (greater than) 10^{-4} .

The behavior of weakly ionized plasmas differs significantly from that of strongly ionized plasmas. In a plasma with a low density of charged particles (i.e., low value of N_e), the effect of the presence of neutral particles overshadows the Coulomb interactions between charged particles. The charged particles collide more often with neutrals than they interact (via the Coulomb repulsion force) with other charged particles, inhibiting collective plasma effects. As the degree of ionization increases, collisions with neutrals become less and less important and Coulomb interactions become increasingly important. In a fully ionized plasma, all particles are subject to Coulomb collisions.

The Sun and the stars are hot enough to be almost completely ionized, with enormous densities ($N_e \simeq 10^{33} \text{ m}^{-3}$), and the interstellar gas is sparse enough to be almost completely ionized as a result of stellar radiation. Starting at about 60 km altitude the Sun bathes our atmosphere in a variety of radiations and the energy in the ultraviolet part of the spectrum is absorbed by atmospheric gas. In the process, significant numbers of air molecules and atoms receive enough energy to become ionized. The resulting free electrons and positive ions constitute the *ionosphere*. Maximum ionization density occurs in the F-region of the ionosphere at about 350 km altitude, where $N_e \simeq 10^{12} \text{ m}^{-3}$. With atmospheric density at 350 km altitude being $N_n \simeq 3.3 \times 10^{14} \text{ m}^{-3}$, the degree of ionization is $\sim 10^{-2}$. At even higher altitudes, the air is thin enough so that it is almost completely ionized, and the motion of charged particles is dominated by the Earth's magnetic field, in a region known as the *magnetosphere*.

Plasmas have various uses in technology because of their unique electrical properties and ability to influence chemical processes. Artificial plasmas are generated by application of heat or strong electric fields. Ultraviolet radiation from plasmas is used in lighting and plasma display panels. The plasma state opens a whole new regime of chemistry not typically accessible to normal gases. Plasmas play a key role in processing materials, including those related to the production of integrated circuits. Plasmas can also be used to process waste, selectively kill bacteria and viruses, and weld materials. Achieving controlled thermonuclear fusion, which holds promise as an abundant and clean energy source, is essentially a plasma physics problem. Thus plasma physics is essential both to understanding the basic processes of our planet and to advancing important technological applications.

One of the most important properties of a plasma is its tendency to remain electrically neutral, i.e., to balance positive and negative free charge ($N_e \simeq N_i$) in any given macroscopic volume element. A slight imbalance in local charge densities gives rise to strong electrostatic forces that act in the direction of restoring neutrality. This property arises from the large charge-to-mass ratio (q_e/m_e) of electrons, so that any significant imbalance of charge gives rise to an electric field of sufficient magnitude to drag a neutralizing cloud of electrons into the positively charged region. If a plasma is subjected to an applied electric field, the free charges adjust so that the major part of the plasma is shielded from the applied field. In order to be considered a plasma, an ionized gas must have a sufficiently large number of charged particles to shield itself electrostatically within a distance smaller than other lengths of physical interest. The quantitative measure of this screening distance is the so-called *Debye length*, discussed below. We will see that the screening distance is proportional to $N_e^{-1/2}$. A simple analogy can be made with a person entering a forest. Beyond a certain distance within the forest there are enough trees to screen the edge of the forest from view. However, if the trees are too far apart and the forest is too small, the person may never lose sight of the edge, in which case such a group of trees would not be called a forest.

At first thought the fourth state of matter may appear to be the simplest to study since the elementary fundamental laws of charged-particle motion are perfectly known, i.e., classical electromagnetic theory (Maxwell's equations) and the Lorentz force equation.² However, analyses of plasma effects are much more

² Gravitational forces are much smaller than electromagnetic forces on earthly scales. The momenta ($p = mv$) of free electrons and ions in typical plasmas are usually high and

complicated, for a number of reasons: (i) Although the various movements of individual particles are all governed by the electromagnetic fields in which they move, these fields are themselves often greatly modified by the presence and motion of the particles. (ii) Atomic processes such as ionization, excitation, recombination, and charge exchange come into play and compete with one another in a complicated manner, with complicated dependencies on particle energies and densities. (iii) The fact that charged particles move results in a variety of transport phenomena arising from both short- and long-range Coulomb interactions between various particles. (iv) The long-range Coulomb forces give rise to a number of collective phenomena, including electrostatic oscillations and instabilities. (v) Most plasmas, and in particular hot plasmas, are typically confined in a magnetic field, with which they are strongly coupled. We may think that in spite of these difficulties it should be possible to solve the equation of motion for each and every particle. We could then find the electric and magnetic fields as functions of space and time by solving Maxwell's equations with the source terms (charged density ρ and current density $\mathbf{J}$) specified using the position and velocity vectors of *all* particles in the system. This type of approach is the domain of the discipline known as computer simulation of plasmas. However, noting that any natural plasma environment (such as the Earth's ionosphere) may contain $>10^{25}$ particles, one has a tremendous accounting problem, with too much information to keep track of.

The fundamental equations governing the behavior of a plasma with freely mobile, non-relativistic particles can be summarized as follows:

$$\left[\begin{array}{c} \text{Initial} \\ \text{and} \\ \text{boundary} \\ \text{conditions} \end{array} \right] \rightarrow \left[\begin{array}{c} \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{D} = \rho \\ \nabla \cdot \mathbf{B} = 0 \\ \mathbf{B} = \mu \mathbf{H} \\ \mathbf{D} = \epsilon \mathbf{E} \end{array} \right] \begin{array}{c} \xrightarrow{\mathbf{E}, \mathbf{B}} \\ \xleftarrow{\mathbf{J}, \rho} \end{array} \left[\begin{array}{c} \frac{d\mathbf{v}_i}{dt} = \frac{q_i}{m_i} (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \\ \rho = \frac{1}{\Delta V} \sum q_i \\ \mathbf{J} = \frac{1}{\Delta V} \sum q_i \mathbf{v}_i \end{array} \right] \leftarrow \left[\begin{array}{c} \text{Initial} \\ \text{conditions} \end{array} \right]$$

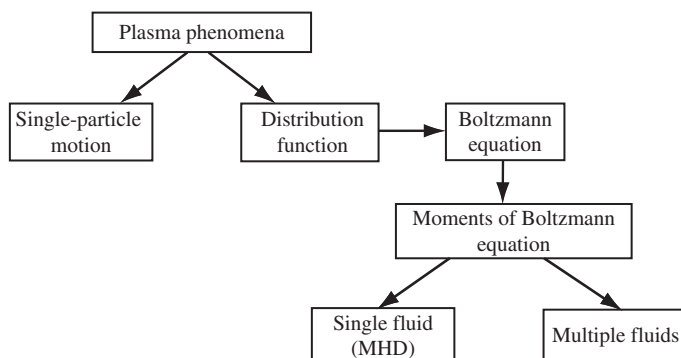
For gaseous plasmas, the medium is essentially free space (aside from freely mobile charged particles, which are separately

their densities relatively low, so that the de Broglie wavelengths ($\lambda_e = h/p$, where $h \simeq 6.6 \times 10^{-34}$ J s is Planck's constant) are much smaller than the mean interparticle distance, so that quantum effects are negligible, except for some types of collisions between particles. As an example, electrons with 1 eV energy have $\lambda_e \simeq 1.2$ nm, while the interparticle distance even for an extremely high electron density of $N_e \simeq 10^{12} \text{ cm}^{-3}$ is $\sim 10^{-4} \text{ cm} = 10^5 \text{ nm} \gg 1.2 \text{ nm}$.

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Figure 1.1 Hierarchy of approaches to plasma phenomena.



accounted for), so that we have $\mu = \mu_0$ and $\epsilon = \epsilon_0$. An equation of motion such as the one above specifying the particle acceleration $d\mathbf{v}/dt$ in terms of the fields $\mathbf{E}$ and $\mathbf{B}$, exists for each and every positively or negatively charged particle in the plasma. For known fields $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ and specified initial conditions, these motion equations can be uniquely solved to determine the positions and velocities of every particle. However, the particle motions and locations lead to charge accumulations and current densities, i.e., ρ and $\mathbf{J}$, which in turn modify the electric and magnetic fields. The charge and current densities are obtained from discrete charges by averaging over a macroscopically small volume ΔV , which nevertheless contains many individual particles, so that it makes sense to talk about continuous distribution of “density” of current and charge. Such averaging is appropriate, since the electric and magnetic fields in Maxwell’s equations are also macroscopic fields, suitably averaged over both space and time.³

Given the complexity of plasma behavior, the field of plasma physics is best described as a web of overlapping models, each based on a set of assumptions and approximations that make a limited range of behavior analytically and computationally tractable. A conceptual view of the hierarchy of plasma models/approaches to plasma behavior that will be covered in this text is shown in Figure 1.1. We will begin with the determination of individual particle trajectories in the presence of electric and magnetic fields. Subsequently, it will be shown that the large number of charged particles in a plasma facilitates the use of statistical techniques

³ Such averaging is assumed to be done over spatial scales that are microscopically large (i.e., contain many individual particles) but which are nevertheless much smaller than any other relevant dimension in the context of a given application, or over time periods much shorter than the resolution of any measuring instrument.

such as plasma kinetic theory, where the plasma is described by a velocity-space distribution function. Quite often, the kinetic-theory approach retains more information than we really want about a plasma and a fluid approach is better suited, in which only macroscopic variables (e.g., density, temperature, and pressure) are kept. The combination of fluid theory with Maxwell's equations forms the basis of the field of magnetohydrodynamics (MHD), which is often used to describe the bulk properties and collective behavior of plasmas.

The remainder of this chapter reviews important physical concepts and introduces basic properties of plasmas.

1.1 Speed, energy, and temperature

The kinetic energy of a particle of mass m moving with a speed u is $E = \frac{1}{2}mu^2$. For an assembly of N particles with different kinetic energies, the average energy E_{av} per particle is given as

$$E_{av} = \frac{1}{2N} \sum_{i=1}^N m_i u_i^2.$$

However, there are other ways of measuring the average energy of an assembly of particles. We will see later that for any gas in thermal equilibrium at temperature T , the average energy per particle is $3k_B T/2$, where $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$ is Boltzmann's constant and T is the absolute temperature. A gas in thermal equilibrium has particles with all speeds, and the most probable distribution of these is the so-called Maxwellian (or Maxwell–Boltzmann) distribution, which in a one-dimensional system is given by

$$f(u) = A e^{-\frac{1}{2} \frac{mu^2}{k_B T}},$$

where $f(u)du$ is the number of particles per unit volume with velocity in the range u to $u + du$. The multiplier A can be determined by noting that the total density N of particles can be found from $f(u)$:

$$N = \int_{-\infty}^{\infty} f(u) du \quad \rightarrow \quad A = N \sqrt{\frac{m}{2\pi k_B T}},$$

where we have used the fact that $\int_{-\infty}^{\infty} e^{-a^2 \xi^2} d\xi = \sqrt{\pi}/a$. The width of the velocity distribution is characterized by the constant

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temperature T . The average kinetic energy of the particles can be calculated from $f(u)$ as follows:

$$E_{\text{av}} = \frac{\int_{-\infty}^{\infty} \frac{1}{2}mu^2 f(u)du}{\int_{-\infty}^{\infty} f(u)du}. \quad (1.1)$$

Upon integration of the numerator by parts, we have $E_{\text{av}} = \frac{1}{2}k_{\text{B}}T$. Extending this result to three dimensions, we find that $E_{\text{av}} = \frac{3}{2}k_{\text{B}}T$, or $\frac{1}{2}k_{\text{B}}T$ per degree of freedom.

With $E_{\text{av}} = \frac{3}{2}k_{\text{B}}T$ a gas at 1 K corresponds to an average energy of 2.07×10^{-23} J per particle. Often it is more convenient to measure particle energy in terms of electron volts (eV). If a particle has an electric charge equal in magnitude to the electronic charge $q_e = -1.602 \times 10^{-19}$ C, and is accelerated through a potential difference of 1 V, it has an energy of 1.602×10^{-19} J. This unit of energy is defined as an electron volt, commonly abbreviated as 1 eV. Thus, to express energy in terms of eV, we must divide the kinetic energy in J by $|q_e| = 1.602 \times 10^{-19}$ C. Hence we have

$$E = \frac{mu^2}{2|q_e|} \text{ eV}.$$

The unit of eV is particularly useful in dealing with charged particles, since it directly indicates the potential necessary to produce a singly charged particle of some particular energy.

It is often convenient to express the temperature of a gas in thermodynamic equilibrium in units of energy (eV). Typically, the energy corresponding to $k_{\text{B}}T$ is used to denote the temperature. Using $k_{\text{B}}T = 1 \text{ eV} = 1.6 \times 10^{-19}$ J, the conversion factor is $1 \text{ eV} = 11\,600 \text{ K}$. Thus, when we refer to a 0.5 eV plasma we mean that $k_{\text{B}}T = 0.5 \text{ eV}$, or a plasma temperature of $T = 5800 \text{ K}$, or an average energy (in three dimensions) of $E_{\text{av}} = \frac{3}{2}k_{\text{B}}T = 0.75 \text{ eV}$. Thus a plasma at a temperature of 300 K has an average energy of 0.0388 eV, and a plasma with 10 keV average energy is at a temperature of $T = 7.75 \times 10^7 \text{ K}$. With reference to our earlier discussion of the plasma state coming into being at sufficiently high temperature for the material to be ionized, the temperature required to form plasmas from pure substances in thermal equilibrium ranges from $\sim 4000 \text{ K}$ for cesium (initially used by Langmuir) to $\sim 20\,000 \text{ K}$ for elements such as helium which are particularly difficult to ionize.

It should be noted that temperature is an equilibrium concept, and we may not always be faced with equilibrium situations. In such cases, a true temperature cannot always be assigned, although we

may still use the term in the sense of average energy. (Note that the average energy can always be calculated for any given distribution using a procedure similar to that given in (1.1).) It should also be noted that by temperature we mean the quantity sometimes called “kinetic temperature,” simply the state of energy of the particles. A high value of T does not necessarily mean a lot of heat, since the latter also depends on heat capacity, determined also by the number of particles. As an example, the electron kinetic temperature inside a fluorescent lamp is $\sim 11\,000$ K but it does not feel nearly as “hot” when one holds the tube while it is lit. The reason is that the free-electron density inside the tube is much less than the number of particles in a gas at atmospheric pressure, so that the total amount of heat transferred to the walls by the impact of electrons is low.

A plasma can have several temperatures at the same time, since often the ions and electrons have separate Maxwellian distributions (of different widths) corresponding to temperatures, respectively, of T_i and T_e . Such equilibria can arise because the collision rate among ions or electrons themselves is larger than that between ions and electrons. Although each species can thus have its own thermal equilibrium, in time the tendency would be for the temperatures to equalize. In a magnetized plasma (i.e., a plasma under the influence of a strong magnetic field), even a single species can have two different temperatures, since the Lorentz forces acting on it along the magnetic field are different than those perpendicular to the field. These different temperatures, typically denoted $T_{e\parallel}$ and $T_{e\perp}$, respectively correspond to Maxwellian distributions of electron velocities along and perpendicular to the magnetic field.

1.2 Quasi-neutrality and plasma oscillations

It was mentioned above that a most fundamental property of a plasma is its tendency to remain electrically neutral and that any small change in local neutrality resulting from charge separation leads to large electric fields which pull electrons back to their original positions. Because of their inertia, the electrons which are pulled back typically oscillate about the initially charged region. However, since this oscillation is typically at a rather high frequency, quasi-neutrality is preserved on a time-average basis.

In this section, we briefly describe the dynamics of this oscillatory behavior of a plasma, which will be studied in detail in later chapters. Consider a steady initial state in which there is a uniform number density $N_e = N_0$ of electrons, neutralized by an equal