SEMICLASSICAL AND STOCHASTIC GRAVITY

The two pillars of modern physics are general relativity and quantum field theory, the former describes the large scale structure and dynamics of space-time, the latter, the microscopic constituents of matter. Combining the two yields quantum field theory in curved space-time, which is needed to understand quantum field processes in the early universe and black holes, such as the well-known Hawking effect. This book examines the effects of quantum field processes back-reacting on the background space-time which become important near the Planck time (10^{-43} sec). It explores the self-consistent description of both space-time and matter via the semiclassical Einstein equation of semiclassical gravity theory, exemplified by the inflationary cosmology, and fluctuations of quantum fields which underpin stochastic gravity, necessary for the description of metric fluctuations (space-time foams). Covering over four decades of thematic development, this book is a valuable resource for researchers interested in quantum field theory, gravitation and cosmology.

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Semiclassical and Stochastic Gravity

Quantum Field Effects on Curved Spacetime

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Dedicated to

The loving memory of my parents, I-Ping Hu and Pie Wang
and my brother, Professor Bambi Hu – BLH

My mother Maria, *my daughter* Lena, *and*
the loving memory of my father Joan *and uncle* Angel – EV

Contents

Preface

Research on the topics covered in this book began around 1974, after Hawking's epochal discovery, when effective action methods were introduced, and regularization schemes established, for curved spacetimes. This book could be viewed as v2 and v3 of "*quantum field theory in curved spacetime*" (QFTCST), established half a century ago, in two senses: as versions 2 and 3, the continued development of this field to "*semiclassical gravity*" in the early 1980s and "*stochastic gravity*" established in the mid-90s, and their many implications and applications in the ensuing years. It can also be viewed as Volumes 2 and 3 of the many well-written books on QFTCST, listed in Chapter 1, comprising the chapters in Parts I–II and Parts III–V, respectively. We see little need to explain the relevance of this subject matter to theoretical physics since it is well indicated in these earlier monographs. Suffice it to say that it is drawn by the allure of quantum gravity, theories for the microscopic structures of spacetime, but is built on the firm and weathered foundation of two well-established theories in the past century: general relativity for the large-scale structures of spacetime and quantum field theory for the description of matter, both valid through experimental and observational tests to an amazingly high degree of accuracy.

This is the long-awaited time to give thanks to those who have influenced our intellectual growth, shaped our professional paths and helped in the writing and editing of this book. BLH wishes to express this: I am deeply indebted to my Ph.D. advisor, the late Professor John A. Wheeler for his guidance, inspiration, patience and understanding, in the tumultuous late 1960s and early 70s, when not only theoretical physics, but also humanistic values and societal priorities were undergoing fundamental changes; to the late Professor Tulio Regge in showing how mathematics can be enjoyed like magic, especially when shown in the ambiance of music from his whistling of Italian opera arias. The highest-caliber scholarship of the late Professors S. Chandrasekhar and Bryce S. DeWitt, and of Professors Stephen L. Adler and Steven Weinberg, has continued to serve as a living standard of perfection for me. Professor Charles W. Misner, from whom I learned differential geometry and whose universe was given to me by Wheeler as a first exercise in theoretical cosmology, and Professor James B. Hartle, from whom I learned the versatile effective action method and many aspects of quantum field theory, are prime examples of how presumably self-absorbed researchers can also be very warm, modest and caring human beings.

EV wishes to acknowledge his colleagues and friends Enrique Álvarez, Jaume Garriga and Renaud Parentani, whose views and insights had a deep influence over the years in his understanding of the different topics discussed in the book.

The raw materials of this book are largely based on papers co-authored with colleagues: James B. Hartle, Stephen A. Fulling, Leonard Parker, Hing-Tong Cho, Paul Anderson and his former Ph.D. student Jason Bates, and work we did with our former Ph.D. students and postdoctorals. We gladly acknowledge their essential contributions: Charis Anastopoulos, Daniel Arteaga, Esteban Calzetta, Roberto Camporesi, Antonio Campos, Ardeshir Eftekharzadeh, Markus Fröb, Chad Galley, Philip Johnson, Rosario Martín, Andrew Matacz, Denjoe O'Connor, Guillem Pérez-Nadal, Juan Pablo Paz, Nicholas Phillips, Stephen Ramsey, Alpan Raval, Albert Roura, Tsung-Chen Shen, Kazutomo Shiokawa, Sukanya Sinha, Chris Stephens, Aris Stylianopoulos and Yuhong Zhang.

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