

SEMICLASSICAL AND STOCHASTIC GRAVITY

The two pillars of modern physics are general relativity and quantum field theory, the former describes the large scale structure and dynamics of space-time, the latter, the microscopic constituents of matter. Combining the two yields quantum field theory in curved space-time, which is needed to understand quantum field processes in the early universe and black holes, such as the well-known Hawking effect. This book examines the effects of quantum field processes back-reacting on the background space-time which become important near the Planck time (10^{-43} sec). It explores the self-consistent description of both space-time and matter via the semiclassical Einstein equation of semiclassical gravity theory, exemplified by the inflationary cosmology, and fluctuations of quantum fields which underpin stochastic gravity, necessary for the description of metric fluctuations (space-time foams). Covering over four decades of thematic development, this book is a valuable resource for researchers interested in quantum field theory, gravitation and cosmology.

BEI-LOK B. HU is Professor of Physics at the University of Maryland. He is a Fellow of the American Physical Society and co-authored *Nonequilibrium Quantum Field Theory* (CUP, 2008). His research in theoretical physics focuses on gravitation and quantum theory.

ENRIC VERDAGUER is Professor of Physics at the University of Barcelona. He is a member of the Royal Astronomical Society, specializes in gravitation and gravitational quantum physics, and is co-author of *Gravitational Solitons* (CUP, 2001).

CAMBRIDGE MONOGRAPHS ON MATHEMATICAL PHYSICS

- S. J. Aarseth *Gravitational N-Body Simulations: Tools and Algorithms*[†]
 D. Ahluwalia *Mass Dimension One Fermions*
 J. Ambjørn, B. Durhuus and T. Jonsson *Quantum Geometry: A Statistical Field Theory Approach*[†]
 A. M. Anile *Relativistic Fluids and Magneto-fluids: With Applications in Astrophysics and Plasma Physics*
 J. A. de Azcárraga and J. M. Izquierdo *Lie Groups, Lie Algebras, Cohomology and Some Applications in Physics*[†]
 O. Babelon, D. Bernard and M. Talon *Introduction to Classical Integrable Systems*[†]
 F. Bastianelli and P. van Nieuwenhuizen *Path Integrals and Anomalies in Curved Space*[†]
 D. Baumann and L. McAllister *Inflation and String Theory*
 V. Belinski and M. Henneaux *The Cosmological Singularity*[†]
 V. Belinski and E. Verdaguer *Gravitational Solitons*[†]
 J. Bernstein *Kinetic Theory in the Expanding Universe*[†]
 G. F. Bertsch and R. A. Broglia *Oscillations in Finite Quantum Systems*[†]
 N. D. Birrell and P. C. W. Davies *Quantum Fields in Curved Space*[†]
 K. Bolejkó, A. Krasinski, C. Hellaby and M-N. Célérier *Structures in the Universe by Exact Methods: Formation, Evolution, Interactions*
 D. M. Brink *Semi-Classical Methods for Nucleus-Nucleus Scattering*[†]
 M. Burgess *Classical Covariant Fields*[†]
 E. A. Calzetta and B.-L. B. Hu *Nonequilibrium Quantum Field Theory*
 S. Carlip *Quantum Gravity in 2+1 Dimensions*[†]
 P. Cartier and C. DeWitt-Morette *Functional Integration: Action and Symmetries*[†]
 J. C. Collins *Renormalization: An Introduction to Renormalization, the Renormalization Group and the Operator-Product Expansion*[†]
 P. D. B. Collins *An Introduction to Regge Theory and High Energy Physics*[†]
 M. Creutz *Quarks, Gluons and Lattices*[†]
 P. D. D'Eath *Supersymmetric Quantum Cosmology*[†]
 J. Dereziński and C. Gérard *Mathematics of Quantization and Quantum Fields*
 F. de Felice and D. Bini *Classical Measurements in Curved Space-Times*
 F. de Felice and C. J. S. Clarke *Relativity on Curved Manifolds*[†]
 B. DeWitt *Supermanifolds, 2nd edition*[†]
 P. G. O. Freund *Introduction to Supersymmetry*[†]
 F. G. Friedlander *The Wave Equation on a Curved Space-Time*[†]
 J. L. Friedman and N. Stergioulas *Rotating Relativistic Stars*
 Y. Frishman and J. Sonnenschein *Non-Perturbative Field Theory: From Two Dimensional Conformal Field Theory to QCD in Four Dimensions*
 J. A. Fuchs *Affine Lie Algebras and Quantum Groups: An Introduction, with Applications in Conformal Field Theory*[†]
 J. Fuchs and C. Schweigert *Symmetries, Lie Algebras and Representations: A Graduate Course for Physicists*[†]
 Y. Fujii and K. Maeda *The Scalar-Tensor Theory of Gravitation*[†]
 J. A. H. Futterman, F. A. Handler, R. A. Matzner *Scattering from Black Holes*[†]
 A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky and E. S. Sokatchev *Harmonic Superspace*[†]
 R. Gambini and J. Pullin *Loops, Knots, Gauge Theories and Quantum Gravity*[†]
 T. Gannon *Moonshine beyond the Monster: The Bridge Connecting Algebra, Modular Forms and Physics*[†]
 A. García-Díaz *Exact Solutions in Three-Dimensional Gravity*
 M. Göckeler and T. Schücker *Differential Geometry, Gauge Theories, and Gravity*[†]
 C. Gómez, M. Ruiz-Altaba and G. Sierra *Quantum Groups in Two-Dimensional Physics*[†]
 M. B. Green, J. H. Schwarz and E. Witten *Superstring Theory Volume 1: Introduction*
 M. B. Green, J. H. Schwarz and E. Witten *Superstring Theory Volume 2: Loop Amplitudes, Anomalies and Phenomenology*
 V. N. Gribov *The Theory of Complex Angular Momenta: Gribov Lectures on Theoretical Physics*[†]
 J. B. Griffiths and J. Podolský *Exact Space-Times in Einstein's General Relativity*[†]
 T. Harko and F. Lobo *Extensions of $f(R)$ Gravity: Curvature-Matter Couplings and Hybrid Metric-Palatini Gravity*
 S. W. Hawking and G. F. R. Ellis *The Large Scale Structure of Space-Time*[†]
 B.-L. Hu and E. Verdaguer *Semiclassical and Stochastic Gravity*
 F. Iachello and A. Arima *The Interacting Boson Model*[†]
 F. Iachello and P. van Isacker *The Interacting Boson-Fermion Model*[†]
 C. Itzykson and J. M. Drouffe *Statistical Field Theory Volume 1: From Brownian Motion to Renormalization and Lattice Gauge Theory*[†]
 C. Itzykson and J. M. Drouffe *Statistical Field Theory Volume 2: Strong Coupling, Monte Carlo Methods, Conformal Field Theory and Random Systems*[†]

- G. Jaroszkiewicz *Principles of Discrete Time Mechanics*
 G. Jaroszkiewicz *Quantized Detector Networks*
 C. V. Johnson *D-Branes*[†]
 P. S. Joshi *Gravitational Collapse and Spacetime Singularities*[†]
 J. I. Kapusta and C. Gale *Finite-Temperature Field Theory: Principles and Applications, 2nd edition*[†]
 V. E. Korepin, N. M. Bogoliubov and A. G. Izergin *Quantum Inverse Scattering Method and Correlation Functions*[†]
 J. Kroon *Conformal Methods in General Relativity*
 M. Le Bellac *Thermal Field Theory*[†]
 L. Lusanna *Non-Inertial Frames and Dirac Observables in Relativity*
 Y. Makeenko *Methods of Contemporary Gauge Theory*[†]
 S. Mallik and S. Sarkar *Hadrons at Finite Temperature*
 A. Malyarenko and M. Ostoja-Starzewski *Tensor-Valued Random Fields for Continuum Physics*
 N. Manton and P. Sutcliffe *Topological Solitons*[†]
 N. H. March *Liquid Metals: Concepts and Theory*[†]
 I. Montvay and G. Münster *Quantum Fields on a Lattice*[†]
 P. Nath *Supersymmetry, Supergravity, and Unification*
 L. O'Raiheartaigh *Group Structure of Gauge Theories*[†]
 T. Ortin *Gravity and Strings, 2nd edition*
 A. M. Ozorio de Almeida *Hamiltonian Systems: Chaos and Quantization*[†]
 M. Paranjape *The Theory and Applications of Instanton Calculations*
 L. Parker and D. Toms *Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity*
 R. Penrose and W. Rindler *Spinors and Space-Time Volume 1: Two-Spinor Calculus and Relativistic Fields*[†]
 R. Penrose and W. Rindler *Spinors and Space-Time Volume 2: Spinor and Twistor Methods in Space-Time Geometry*[†]
 S. Pokorski *Gauge Field Theories, 2nd edition*[†]
 J. Polchinski *String Theory Volume 1: An Introduction to the Bosonic String*[†]
 J. Polchinski *String Theory Volume 2: Superstring Theory and Beyond*[†]
 J. C. Polkinghorne *Models of High Energy Processes*[†]
 V. N. Popov *Functional Integrals and Collective Excitations*[†]
 L. V. Prokhorov and S. V. Shabanov *Hamiltonian Mechanics of Gauge Systems*
 S. Raychaudhuri and K. Sridhar *Particle Physics of Brane Worlds and Extra Dimensions*
 A. Recknagel and V. Schiomerus *Boundary Conformal Field Theory and the Worldsheet Approach to D-Branes*
 M. Reuter and F. Saueressig *Quantum Gravity and the Functional Renormalization Group*
 R. J. Rivers *Path Integral Methods in Quantum Field Theory*[†]
 R. G. Roberts *The Structure of the Proton: Deep Inelastic Scattering*[†]
 P. Romatschke and U. Romatschke *Relativistic Fluid Dynamics In and Out of Equilibrium: And Applications to Relativistic Nuclear Collisions*
 C. Rovelli *Quantum Gravity*[†]
 W. C. Saslaw *Gravitational Physics of Stellar and Galactic Systems*[†]
 R. N. Sen *Causality, Measurement Theory and the Differentiable Structure of Space-Time*
 M. Shifman and A. Yung *Supersymmetric Solitons*
 Y. M. Shnir *Topological and Non-Topological Solitons in Scalar Field Theories*
 H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers and E. Herlt *Exact Solutions of Einstein's Field Equations, 2nd edition*[†]
 J. Stewart *Advanced General Relativity*[†]
 J. C. Taylor *Gauge Theories of Weak Interactions*[†]
 T. Thiemann *Modern Canonical Quantum General Relativity*[†]
 D. J. Toms *The Schwinger Action Principle and Effective Action*[†]
 A. Vilenkin and E. P. S. Shellard *Cosmic Strings and Other Topological Defects*[†]
 R. S. Ward and R. O. Wells, Jr *Twistor Geometry and Field Theory*[†]
 E. J. Weinberg *Classical Solutions in Quantum Field Theory: Solitons and Instantons in High Energy Physics*
 J. R. Wilson and G. J. Mathews *Relativistic Numerical Hydrodynamics*[†]

[†] Available in paperback

Cambridge University Press
978-0-521-19357-3 — Semiclassical and Stochastic Gravity
Bei-Lok B. Hu , Enric Verdaguer
Frontmatter
[More Information](#)

Semiclassical and Stochastic Gravity

Quantum Field Effects on Curved Spacetime

BEI-LOK B. HU
University of Maryland, College Park

ENRIC VERDAGUER
University of Barcelona



Cambridge University Press
978-0-521-19357-3 – Semiclassical and Stochastic Gravity
Bei-Lok B. Hu , Enric Verdaguer
Frontmatter
[More Information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.
It furthers the University's mission by disseminating knowledge in the pursuit of
education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521193573
DOI: 10.1017/9780511667497

© Bei-Lok B. Hu and Enric Verdaguer 2020

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2020

Printed in the United Kingdom by TJ International Ltd, Padstow Cornwall
A catalogue record for this publication is available from the British Library.

ISBN 978-0-521-19357-3 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy
of URLs for external or third-party internet websites referred to in this publication
and does not guarantee that any content on such websites is, or will remain,
accurate or appropriate.

Dedicated to

*The loving memory of my parents, I-Ping Hu and Pie Wang
and my brother, Professor Bambi Hu – BLH*

*My mother Maria, my daughter Lena, and
the loving memory of my father Joan and uncle Angel – EV*

Cambridge University Press
978-0-521-19357-3 — Semiclassical and Stochastic Gravity
Bei-Lok B. Hu , Enric Verdaguer
Frontmatter
[More Information](#)

Contents

<i>Preface</i>	<i>page</i> xiii
1 Overview: Main Themes. Key Issues. Reader's Guide	1
1.1 From QFT in Curved Spacetime to Semiclassical and Stochastic Gravity	2
1.2 Quantum, Stochastic, Semiclassical	15
1.3 Low Energy Limit: Relation to Gravitational Quantum Physics	20
1.4 Emphasis and Approach. Guide to the Reader	28
 Part I Effective Action and Regularization, Stress Tensor and Fluctuations	
2 'In-Out' Effective Action. Dimensional Regularization	37
2.1 Quantum Field Theory in Dynamical Spacetimes: Key Points	38
2.2 The Schwinger–DeWitt ('In-Out') Effective Action	46
2.3 Effective Action of an Interacting Field: Particle Creation and Interaction	56
2.4 Quasilocal Effective Action for Slowly Varying Background	64
2.5 Regularization of Quasilocal Lagrangian for $\lambda\Phi^4$ Field	71
2.6 Renormalization Group Equations	75
3 'In-In' Effective Action. Stress Tensor. Thermal Fields	79
3.1 The 'In-In' Effective Action	79
3.2 'In-In' Formalism for Quantum Fields in Curved Spacetime	87
3.3 In-In Effective Action in Bianchi Type I Universe	89
3.4 Expectation Value of the Stress Energy Tensor for Interacting Fields	101
3.5 CTP Effective Action for Thermal Fields	106
4 Stress-Energy Tensor and Correlators: Zeta-Function Method	113
4.1 Zeta Function Regularization of 1-loop Effective Potential	114
4.2 One-Loop Finite Temperature Effective Potential	118
4.3 One-Loop Effective Potential in the Einstein Universe	121
4.4 $O(N)$ Self-Interacting Scalar Field in Curved Spacetime	126
4.5 Stress-Energy 2-Pt Function from 2nd Variation of Effective Action	133
4.6 Energy Density Fluctuations in $\Sigma = R^d \times S^1$	140
4.7 Correlations of the Stress-Energy Tensor in AdS Space	144

x	<i>Contents</i>	
5	Stress-Energy Tensor and Correlation. Point Separation	150
5.1	Stress-Energy Bitensors and Products	151
5.2	Point-Separation Regularization of the Stress-Energy Tensor	160
5.3	The Noise Kernel: Structure, Forms and Computations	169
Part II Infrared Behavior, 2PI, I/N, Backreaction and Semiclassical Gravity		
6	Infrared Behavior of Interacting Quantum Fields	185
6.1	Overview: Relevance, Issues and Approaches	185
6.2	Euclidean Zero-Mode, EIRD, 2PI Effective Action	196
6.3	Lorentzian de Sitter: Late Time IR and Stochastic Approach	207
6.4	Nonperturbative RG. Graviton and Gauge Issues	221
7	Advanced Field Theory Topics	228
7.1	2PI CTP Effective Action in Curved Spacetime	229
7.2	The $O(N)$ Field Theory in Curved Spacetime	240
7.3	Remark: Consistent Renormalization of 2PI Effective Action	252
7.4	Solving the Gap Equation for the Infrared Behavior of $O(N)$ Field in dS	253
7.5	Yukawa Coupled Scalar and Spinor Fields in Curved Spacetime	255
8	Backreaction of Early Universe Quantum Processes	265
8.1	Vacuum Energy-Driven Cosmology	266
8.2	Backreaction of Cosmological Particle Creation	283
8.3	Preheating from Inflaton Particle Creation and Interaction	295
8.4	Other Examples: Stochastic Inflation, Minisuperspace Cosmology	311
Part III Stochastic Gravity		
9	Metric Correlations at One-Loop: In-In and Large N	317
9.1	The In-In Formalism in Flat Spacetime	318
9.2	The In-In or CTP Effective Action	324
9.3	Gravity and Matter Interaction in the CTP Formalism	327
9.4	Large N Expansion: A Toy Model	334
10	The Einstein–Langevin Equation	337
10.1	Semiclassical Gravity: Axiomatic Approach	338
10.2	Stochastic Gravity: Axiomatic Approach	342
10.3	Validity of Semiclassical Gravity	349
10.4	Functional Approach	354
10.5	Explicit Form of the Einstein–Langevin Equation	360

11 Metric Fluctuations in Minkowski Spacetime	364
11.1 Perturbations around Minkowski Spacetime	365
11.2 The Kernels in the Minkowski Background	367
11.3 Einstein–Langevin Equation	371
11.4 Solutions of the Einstein–Langevin Equation	374
11.5 Stability of Minkowski Spacetime	381
Part IV Cosmological and Black Hole Backreaction with Fluctuations	
12 Cosmological Backreaction with Fluctuations	391
12.1 The Backreaction Problem in Cosmology	392
12.2 Influence Action for Cosmological Perturbations	394
12.3 Einstein–Langevin Equation	398
12.4 Detailed Computation of the Trace Terms	401
12.5 Mathematical Supplement	405
13 Structure Formation in the Early Universe	410
13.1 The Model	411
13.2 Einstein–Langevin Equation for Scalar Perturbations	412
13.3 Correlation for Scalar Perturbations	416
13.4 Equivalence of the Stochastic–Quantum Correlations	418
13.5 Including One-Loop Contributions	421
14 Black Hole Backreaction and Fluctuations	423
14.1 Issues, Proposals and Scenarios	423
14.2 General Issues on Backreaction	434
14.3 Backreaction under Quasi-Static Conditions	439
14.4 Metric Fluctuations of an Evaporating Black Hole	452
14.5 Work on Metric Fluctuations without Backreaction	462
Part V Quantum Curvature Fluctuations in de Sitter Spacetime	
15 Stress-Energy Tensor Fluctuations in de Sitter Space	467
15.1 De Sitter Geometry and Invariant Bitensors	468
15.2 Noise Kernel in de Sitter Spacetime	473
15.3 Analysis Based on Field Modes	477
15.4 Implications for Gravitational Fluctuations	480
16 Two-Point Metric Perturbations in de Sitter	483
16.1 Effective Action for Cosmological Perturbations	484
16.2 Classification of Metric Perturbations	489

16.3	Two-Point Functions for Tensor Metric Perturbations	494
16.4	Intrinsic/Induced Fluctuations and Secular Terms	501
16.5	Two-Point Functions for Metric Perturbations	505
16.6	Effective Action for a General Conformal Field Theory	509
16.7	Mathematical Supplement	513
17	Riemann Tensor Correlator in de Sitter	519
17.1	De Sitter-Invariant Bitensors	520
17.2	Correlators of Curvature Tensors	522
17.3	Riemann Tensor Correlator for General CFTs	533
17.4	Riemann Tensor Correlator in Minkowski Spacetime	536
17.5	Useful Fourier Transforms	538
18	Epilogue: Linkage with Quantum Gravity	540
18.1	A New Perspective and Two Different Routes	540
18.2	Emergent versus Quantum Gravity	542
18.3	Unraveling the Microstructures of Spacetime	544
18.4	Relation to Quantum Gravity and Limitations of Stochastic Gravity	547
	<i>References</i>	550
	<i>Index</i>	591

Preface

Research on the topics covered in this book began around 1974, after Hawking's epochal discovery, when effective action methods were introduced, and regularization schemes established, for curved spacetimes. This book could be viewed as v2 and v3 of "*quantum field theory in curved spacetime*" (QFTCST), established half a century ago, in two senses: as versions 2 and 3, the continued development of this field to "*semiclassical gravity*" in the early 1980s and "*stochastic gravity*" established in the mid-90s, and their many implications and applications in the ensuing years. It can also be viewed as Volumes 2 and 3 of the many well-written books on QFTCST, listed in Chapter 1, comprising the chapters in Parts I–II and Parts III–V, respectively. We see little need to explain the relevance of this subject matter to theoretical physics since it is well indicated in these earlier monographs. Suffice it to say that it is drawn by the allure of quantum gravity, theories for the microscopic structures of spacetime, but is built on the firm and weathered foundation of two well-established theories in the past century: general relativity for the large-scale structures of spacetime and quantum field theory for the description of matter, both valid through experimental and observational tests to an amazingly high degree of accuracy.

This is the long-awaited time to give thanks to those who have influenced our intellectual growth, shaped our professional paths and helped in the writing and editing of this book. BLH wishes to express this: I am deeply indebted to my Ph.D. advisor, the late Professor John A. Wheeler for his guidance, inspiration, patience and understanding, in the tumultuous late 1960s and early 70s, when not only theoretical physics, but also humanistic values and societal priorities were undergoing fundamental changes; to the late Professor Tulio Regge in showing how mathematics can be enjoyed like magic, especially when shown in the ambiance of music from his whistling of Italian opera arias. The highest-caliber scholarship of the late Professors S. Chandrasekhar and Bryce S. DeWitt, and of Professors Stephen L. Adler and Steven Weinberg, has continued to serve as a living standard of perfection for me. Professor Charles W. Misner, from whom I learned differential geometry and whose universe was given to me by Wheeler as a first exercise in theoretical cosmology, and Professor James B. Hartle, from whom I learned the versatile effective action method and many aspects of quantum field theory, are prime examples of how presumably self-absorbed researchers can also be very warm, modest and caring human beings.

EV wishes to acknowledge his colleagues and friends Enrique Álvarez, Jaume Garriga and Renaud Parentani, whose views and insights had a deep influence over the years in his understanding of the different topics discussed in the book.

The raw materials of this book are largely based on papers co-authored with colleagues: James B. Hartle, Stephen A. Fulling, Leonard Parker, Hing-Tong Cho, Paul Anderson and his former Ph.D. student Jason Bates, and work we did with our former Ph.D. students and postdoctorals. We gladly acknowledge their essential contributions: Charis Anastopoulos, Daniel Arteaga, Esteban Calzetta, Roberto Camporesi, Antonio Campos, Ardeshir Eftekharzadeh, Markus Fröb, Chad Galley, Philip Johnson, Rosario Martín, Andrew Matacz, Denjoe O'Connor, Guillem Pérez-Nadal, Juan Pablo Paz, Nicholas Phillips, Stephen Ramsey, Alpan Raval, Albert Roura, Tsung-Chen Shen, Kazutomo Shiokawa, Sukanya Sinha, Chris Stephens, Aris Stylianopoulos and Yuhong Zhang.

After the drafts of the chapters were produced an important role was played by the chapter readers, who made very helpful suggestions for improvements. For this we wish to give special thanks to Professors Paul Anderson, Hing-Tong Cho, Jen-Tsung Hsiang, Diego Mazzitelli, Shun-Pei Miao, Diana López Nacir and Albert Roura for their devoted assistance, often down to checking the consistency of notations. A note of appreciation goes to Professor Chong-Sun Chu for his keen interest and the useful suggestion of adding a subtitle to the book.

Finally, we wish to thank Simon Capelin, senior editor of Cambridge University Press, for his sustained interest and patience, and Sarah Lambert for her helpful advice in the editing and production of this book.

On a personal note, BLH is grateful for the unfailing support of his brother, Professor Shiu-Lok Hu, his cousin Kuen-Wai Lau and wife Alice Cho, and lifelong friend Dr. Shun-Ming Chung and his endearing family. He misses even more deeply his children Tung-Hui and Tung-Fei, his love in an eternal universe, even after all vital signals have disappeared in a black hole.

B. L. Hu, College Park, USA

E. Verdaguer, Barcelona, ES